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## Use of Optimal Controllers for Multidimensional Technological Objects

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### ABSTRACT

We study multidimensional optimal controllers for technological objects on an example of a subsystem of controlling temperature of juice-shaving mixture in an oblique diffusive plant of sugar production. On the basis of computer simulation we determine conditions, advantages and disadvantages of each of the systems with different controllers for the given object. Variants of implementation of some algorithms on modern technical automation means are given.

**Key words:** optimal control, sugar production, oblique diffusive plant, juice-shaving mixture, transient processes, random perturbation, computer simulation.

In automation of multidimensional technological objects we often face the problem of stabilization of technological regime, which results in necessity of controlling several interdependent variables. Presently this problem is solved in two ways: either with using several automatic controllers for each variable or by creating one multidimensional controller.

This paper deals with issues of using multidimensional controllers for complex technological objects, including linearly-quadratic with Gaussian perturbations (LQG),  $H_2$ - and  $H_\infty$ -controllers. Selection of these controllers is conditioned by common features of mathematical apparatus of their syntheses and using mathematical models in the state space.

Recently there have been published many papers related to the mentioned subject field, mainly in the field of aerospace industry. Therefore in this paper we omit foundations of mathematical apparatus (they can be found in [1–3]) and pay main attention to possibilities of use of the mentioned controllers for continuous technological objects, their comparison in modeling, and also to problems arising in the course of implementation.

Development of automation of objects of continuously-running fabrication, in particular, food industry, follows the way of implementing microprocessor technology, and the main attention is paid to information support, not implementation of new methods and controlling algorithms. This is caused by technological requirements to modern automation systems and also with the fact, that tracking process and decision making is still committed to operator. Besides, using new algorithms is associated with laborious work, selection of additional software, writing and debugging program, maintenance of working capacity within a lengthy working interval, which result in increasing its cost and is not paid off. Therefore in enterprises with continuously-running fabrication, implementation of automation systems is restricted, as a rule, by using single-circuit controlling systems (PI-regulators), logic control, multiloop systems and some adaptive algorithms, for example, for tuning of controllers.

Let us consider a class of objects with  $n$  controlled variables, for which using a single multidimensional controller is expedient.

The first stage in synthesis of multidimensional controllers is selection of object and assessment of peculiarities of control. On the basis of theoretical and experimental data we concluded, that such controllers are expedient in application for technological objects with regulation of interdependent parameters. Therefore, a class of objects with the following characteristics [4] is singled out.

- Let us have  $n \geq 4$  coordinates of state of the same physical nature. For smaller number of state coordinates, it is recommended to use simpler controllers. The upper bound is not defined, however increasing dimension of the state vector increases dimension of controller and speed of recalculation of its parameters, which in real time can result in significant delay.
- There are internal interconnections between variables. We should note that this interconnection should be essential and mutual, for example: one coordinate of state influences a larger number of object variables. Besides, we should take into account, that disproportion of coefficients of state coordinates interconnection affects scattering of controller coefficients.
- Mathematical model is described by equations of the same type, i.e., in derivation of equations the same logical approach is used.
- Existing local controlling systems are analogous.

To this class of objects belong several subsystems of food industry, for example, the subsystem of controlling levels in a multiple-effect evaporating plant of sugar factory and collectors before and after it, subsystem of temperature regulating in an oblique diffusive plant (ODP). From this point onwards in the role of a controlled object we shall consider the subsystem of temperature regulating of ODP.

Since each LQG,  $H_2$ - and  $H_\infty$ -optimal controller is based on a mathematical model, represented in the state space, the second stage of synthesis is derivation of the model of the selected object.

On the basis of heat balances in view of some assumptions we derived a linear mathematical model of subsystem of ODP by temperatures (computation time  $\tau = 100t$ , s):

$$\begin{aligned}
1.157 \frac{d\Delta\theta_1}{dt} + \Delta\theta_1 &= 0.43\Delta\theta_c + 0.54\Delta\theta_2 + 0.03\Delta\theta_{n1} - 0.27\Delta G_c + 0.15\Delta G_{dc}; \\
1.169 \frac{d\Delta\theta_2}{dt} + \Delta\theta_2 &= 0.43\Delta\theta_1 + 0.54\Delta\theta_3 + 0.03\Delta\theta_{n2} - 0.14\Delta G_c + 0.04\Delta G_{dc}; \\
1.181 \frac{d\Delta\theta_3}{dt} + \Delta\theta_3 &= 0.43\Delta\theta_2 + 0.54\Delta\theta_4 + 0.03\Delta\theta_{n3} - 0.04\Delta G_c + 0.03\Delta G_{dc}; \\
1.193 \frac{d\Delta\theta_4}{dt} + \Delta\theta_4 &= 0.43\Delta\theta_3 + 0.33\Delta\theta_b + 0.03\Delta\theta_{n4} + 0.21\Delta\theta_g + 0.92\Delta G_b + \\
&\quad + 0.92\Delta G_g - 0.87\Delta G_{dc} + 0.09\Delta G_c;
\end{aligned} \tag{1}$$

$$0.0887 \frac{d\Delta\theta_{n1}}{dt} + \Delta\theta_{n1} = \Delta\theta_1 + 279.91\Delta G_{n1};$$

$$0.0393 \frac{d\Delta\theta_{n2}}{dt} + \Delta\theta_{n2} = \Delta\theta_2 + 288.9\Delta G_{n2};$$

$$0.0489 \frac{d\Delta\theta_{n3}}{dt} + \Delta\theta_{n3} = \Delta\theta_3 + 286.5\Delta G_{n3};$$

$$0.0715 \frac{d\Delta\theta_{n4}}{dt} + \Delta\theta_{n4} = \Delta\theta_4 + 282.3\Delta G_{n4}.$$

The formulated problem is solved using a mathematical model in the state space

$$\frac{dx}{dt} = Ax(t) + Bu(t) + Gw(t); \tag{2}$$

$$y_v = Cx(t) + Du(t) + Hw(t),$$

where vectors of model

$$x = [\Delta\theta_1, \Delta\theta_2, \Delta\theta_3, \Delta\theta_4, \Delta\theta_{n1}, \Delta\theta_{n2}, \Delta\theta_{n3}, \Delta\theta_{n4}]^T \tag{3}$$

constitute vector of state coordinates, which is determined by temperatures of juice-shaving mixture and steam in steam chests of four zones of the apparatus respectively;  $u$ , the control vector, is the steam consumption in zones of apparatus,  $w$  is the vector of perturbations,

$$u = [\Delta G_{n1}, \Delta G_{n2}, \Delta G_{n3}, \Delta G_{n4}]^T; \tag{4}$$

$$w = [\Delta\theta_c, \Delta G_c, \Delta G_{dc}, \Delta\theta_b, \Delta\theta_g, \Delta G_b, \Delta G_g]^T, \tag{5}$$

where  $\Delta G_c, \Delta G_{dc}, \Delta G_b, \Delta G_g$  are expenditures of shavings, diffusion juice, barometric and bagasse water respectively,  $\Delta\theta_c, \Delta\theta_b, \Delta\theta_g$  are temperatures of shavings at inlet to diffusive apparatus, of barometric and bagasse respectively;  $y_v$  is the vector of outputs (measurements), temperatures of juice-shaving mixture in the corresponding zones of apparatus,

$$y_v = [\Delta\theta_1, \Delta\theta_2, \Delta\theta_3, \Delta\theta_4]^T. \quad (6)$$

Matrices of constant coefficients  $A, B, C, D, G, H$  in (2) are derived on the basis of model (1).

At the third stage, in accordance with the theory of synthesis of LQG,  $H_2$ - and  $H_\infty$ -controllers, we calculate parameters  $A_c, B_c, C_c$  of optimal multidimensional controller with the structure

$$\begin{aligned} \frac{dx_c(t)}{dt} &= A_c x_c(t) + B_c y_v(t); \\ u(t) &= C_c x_c(t). \end{aligned} \quad (7)$$

For simulation and comparison of LQG,  $H_2$ - and  $H_\infty$ -controllers we take the following conditions of perturbations variation. On the basis of industrial data we performed statistical analysis of variables of the selected object and concluded, that all perturbations of the object are random signals, however, periodically there arise regular situations, when perturbations have a deterministic component. Therefore in simulation we selected the following signals as perturbations:

- white noise;
- random signal, distributed by the normal law;
- deterministic signals;
- random signal, containing a deterministic component.

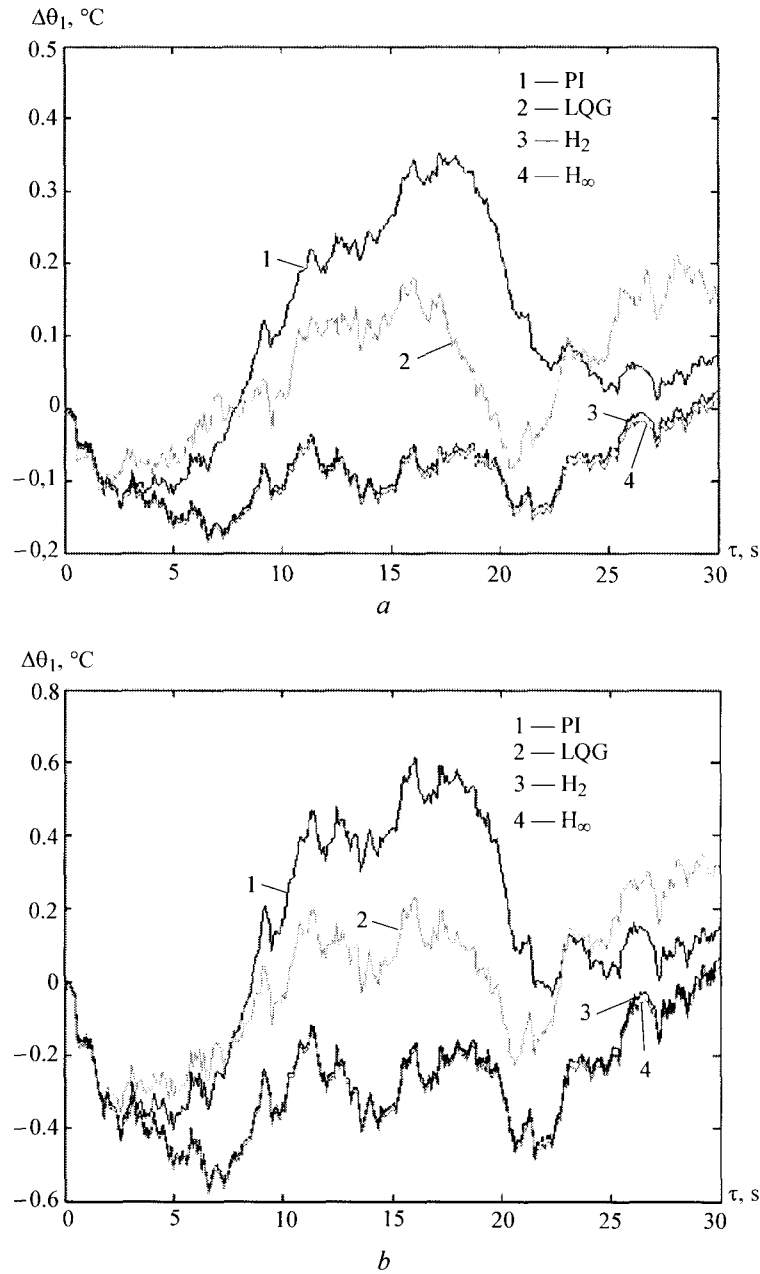
Besides that, simulation is performed for the following cases:

- under nominal characteristics of perturbations, i.e., in the form of white noise and a random signal with deviation, calculated by statistical analysis of actual object and a deterministic signal with 20 % exceeding of nominal;
- under nominal/changed characteristics of perturbations and (or) nominal/changed parameters of mathematical model (20 %).

We also account for random additive signal (white noise) by measurement channels, calculated in statistical analysis, and random multiplicative signal of small intensity.

Results of computer simulation are presented in Figures 1–3, as perturbations we used random processes, reflecting change of load (vector  $w(t)$ ). As the criterion of assessment of control performance and comparison of different controllers we took deviation of transient processes in stabilization of temperatures of juice-shaving mixture.

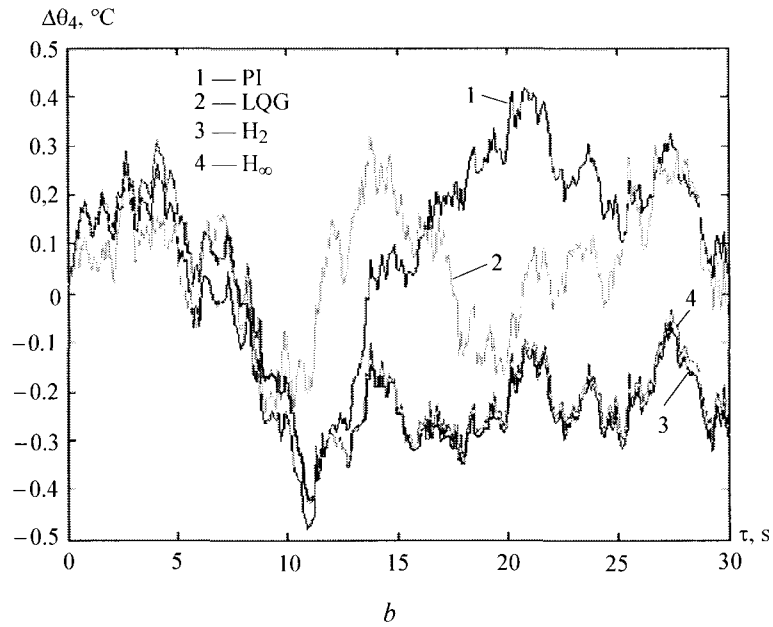
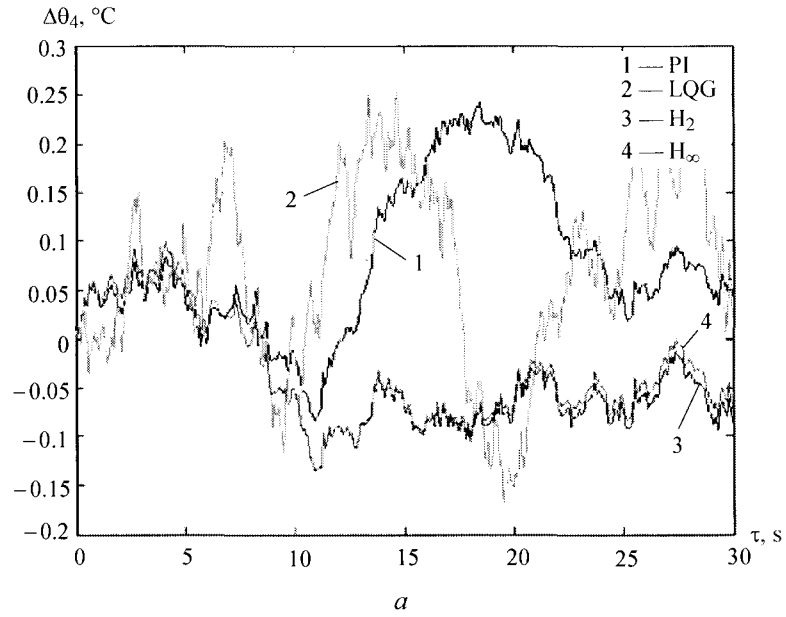
Under perturbation in the form of white noise (Figure 1, *a*, nominal, *b*, changed characteristics of perturbations) better transient processes are observed in the control system with  $H_2$ - and  $H_\infty$ -controllers, here  $H_2$ -controller has better indices of quality of transient processes both in the case with nominal characteristics of perturbation and in the case with changed ones.



**Figure 1**

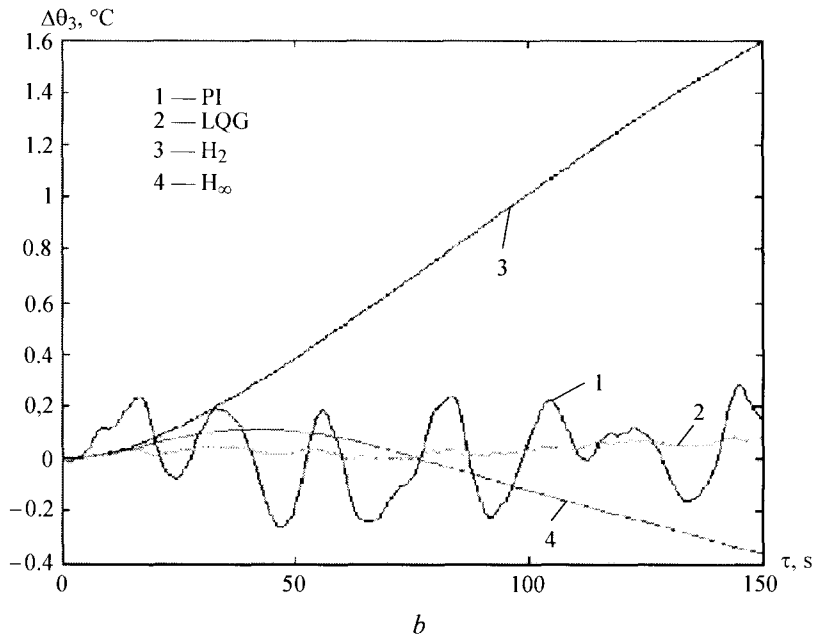
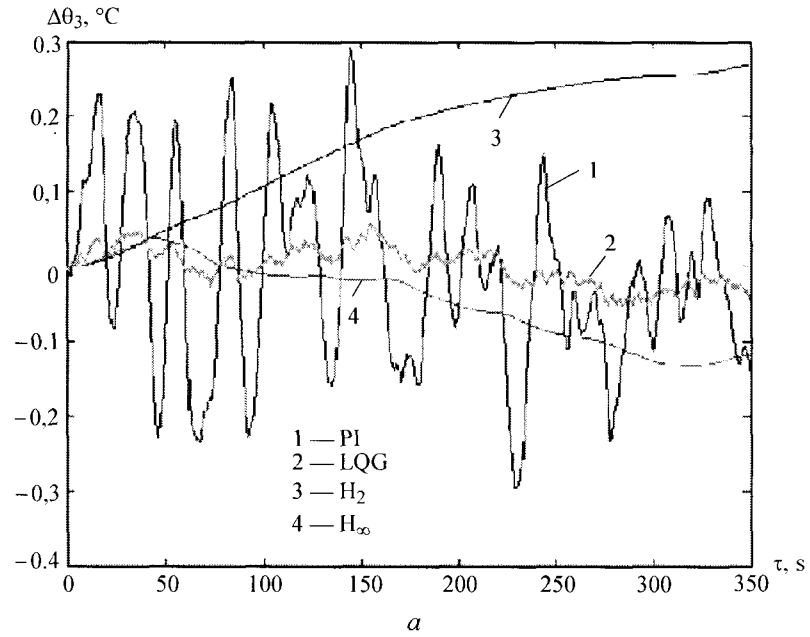
Under perturbations in the form of a random signal with normal distribution (Figure 2, *a*, normal, *b*, changed characteristics of perturbations) we observe the same dependencies.

Under deterministic perturbation and a random perturbation, containing a deterministic component (Figure 3, *a*, deterministic perturbation, *b*, perturbation in the form of a random signal containing a deterministic component) situation changes: in a system with PI- and LQG-controllers, transient processes have random (oscillatory) character at the expense of essential noise in measurement channels; in a system with  $H_2$ - and  $H_\infty$ -controllers, transient processes are distinguished by a significant statistical error, smaller oscillations, in which certain roughness (robustness) of the system is displayed.



**Figure 2**

When accounting for nonlinearities in an automatic control system with perturbations in the form of white noise we observe worsening quality of transient processes (in some cases divergent), which requires changing settings of local controllers. The best remains the system with  $H_2$ -controller, and the control signal here is minimal in a system with  $H_\infty$ -controller in accordance with the minimization criterion. Permanent moves of the controlling organ in a system with LQG-regulator result in increasing of steam consumption, which reduces economic effect in control. The system with  $H_2$ -controller has an advantage compared to the system with PI-controller in saving energy resources.



**Figure 3**

Under stepwise perturbations taking into account nonlinearities, transient processes in a system with  $H_2$ - and  $H_\infty$ -controllers have statistical error; it is smaller in  $H_\infty$ -system. Relationship of control performance is the same, as in the previous case.

Under random perturbations, containing deterministic component with account for nonlinearities, better transient processes, however higher energy consumption are characteristic for a system with LQG-controller. Here, transient processes with  $H_2$ - and  $H_\infty$ -controllers have statistical errors, though their energy consumption is minimal.

By results of simulation we established conditions, advantages and disadvantages of each system with different controllers (Table).

Application of controllers (7) for the selected class of objects requires use of digital algorithms, which is implemented in modern microprocessor systems in different ways: in microcontrollers (single-crystal microcontrollers), in IBM-compatible controllers with the aid of programming languages C, C++, Pascal, Assembler, Basic (controllers ICP DAS of company ICP CON, ADAM of company Advantech), in IBM-compatible controllers with built-in kernel Soft Logic (ISA GRAF, Trace Mode, LabView), in PLC (programmable logical controllers), in any technical means with use of a personal computer (PC) where SCADA-program or (and) an applied software is functioning, in PC with built-in functions of control and SCADA, in mentioned technical means including database control systems. In selection of hardware and software means we also take into account their inertia, flexibility, availability of a technological programming language and graphic interface etc. Besides, we should take into account level of automation and technical means of a specific technological complex, which enables us to built-in the considered algorithms into the existing system instead of creation of a separate automation module of selected subsystems.

**Table**

Controllers	Conditions	Advantages	Disadvantages
LQG	Perturbation is white noise with certain characteristics, mathematical model clearly describes process	Worsening qualitative characteristics of transient processes both under random and under deterministic perturbations	High energy expenses
$H_2$	Perturbation is white noise, parameters of mathematical model can change in working range	Improving qualitative characteristics of transient processes under random perturbations, minimal expenditures for control	Worsening qualitative characteristics of transient processes under deterministic perturbations
$H_\infty$	Perturbation is deterministic, mathematical model is additively and (or) mutiplicatively perturbed	Improving qualitative characteristics of transient processes both under random and under deterministic perturbations, minimal expenditures for control	Slow transient processes under deterministic perturbations

Thus, on the basis of results of this paper we can conclude the following:

1. There exist objective conditions of possibility of application of multidimensional optimal controllers (LQG,  $H_2$ ,  $H_\infty$ ) for technological objects with interdependent coordinates, working under conditions of random perturbations.

2. Simulation of systems with these controllers demonstrated possibility of reduction of the total error of control.

3. Modern technical means enable one to implement mentioned controllers within the framework of an automated system of controlling technological complexes.

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