МІНІСТЕРСТВО ОСВІТИ І НАУКИ УКРАЇНИ

КІРОВОГРАДСЬКИЙ НАЦІОНАЛЬНИЙ ТЕХНІЧНИЙ УНІВЕРСИТЕТ

ЗБІРНИК НАУКОВИХ ПРАЦЬ
КІРОВОГРАДСЬКОГО НАЦІОНАЛЬНОГО ТЕХНІЧНОГО УНІВЕРСИТЕТУ

ТЕХНІКА В СІЛЬСЬКОГОСПОДАРСЬКОМУ ВИРОБНИЦТВІ, ГАЛУЗЕВЕ МАШИНОБУДУВАННЯ, АВТОМАТИЗАЦІЯ

Випуск 30

КРОПИВНИЦЬКИЙ -2017

УДК 681.513.5

Oleksiy Lobok, Assoc. PhD sci., Boris Goncharenko, Prof., PhD tech. sci.,
National University of Food Technologies, Kyiv, Ukraine
Larisa Vihrova, Prof., PhD tech. sci.,
Kirovograd National Technical University, Kropyvnytskyi, Ukraine
E-mail: VIHROVALG@ukr.net

OPTIMAL CONTROL OF LINEAR DYNAMIC DISTRIBUTED SYSTEMS UNDER UNCERTAINTY

Abstract: The article considers the problems of synthesis of optimal control systems that operate in conditions of an uncertain information and are described by generalized equations in partial derivatives of parabolic type. Control has the form of feedback from the observed measurements for the implementation of which it is necessary to solve integral-differential equation of Riccati. Separately built distributed and concentrated limiting regulators and are recursive algorithm for determining the optimal control regarding changes in the number of observations. There is an algorithm designed for determining the required number of point regulators and their optimal location on the border of the field in which the quality criterion does not exceed a specified threshold.

Keywords: Minimax control, point boundary regulators, Sobolewski spaces, inequality of Rayleigh, bilinear form.

А.П Лобок, доц., канд. техн. наук, Б.Н. Гончаренко, проф., докт. техн. наук

Национальный университет пищевых технологий, Киев, Украина

Л.Г. Вихрова, проф., канд. техн. наук.

Кировоградский национальный технический университет, Кропивницкий, Украина

Оптимальное управление линейными динамическими распределенными системами в условиях неопределенности

В статье рассмотрены задачи синтеза оптимального управления системами, функционирующими в условиях неопределенной информации и описываются обобщенными уравнениями в частных производных параболического типа. Управление имеет вид обратной связи от наблюдаемых измерений, для реализации которого необходимо решить интегро-дифференциальное уравнение типа Риккати. Отдельно построены, распределены и сосредоточены предельные регуляторы, а также приведен рекуррентный алгоритм определения оптимального управления по изменению числа наблюдений. Разработан алгоритм определения необходимого количества точечных регуляторов и их оптимальное расположение на границе области, при которых критерий качества не превышает заданного порогового значения.

Ключевые слова: минимаксное управление, точечные предельные регуляторы, Соболевские пространства, неравенство Рэлея, билинейная форма.

INTRODUCTION

To ensure high quality of systems of regulation, it is necessary to use more precise mathematical models of control objects that take into account not only the time but also the spatial coordinates namely systems with distributed parameters. It is necessary to consider the problem of constructing regulators for the class of systems with distributed parameters of parabolic type, to find a constructive solution to the problem of minimax synthesis boundary distributed and point control, also to find algorithm to determine the number and the optimal location of point regulators.

STATEMENT OF THE PROBLEM AND ANALYSIS OF RECENT RESEARCHES

Tasks of minimax control for systems with lumped parameters are operating under conditions of uncertainty considered in [1, 2]. Using the methods of perturbation theory in [3, 4] we receive the solution of these problems for systems with distributed parameters with more general functions of value. There is conducted further development of the theory of minimax controlling with regard to systems with distributed parameters described by generalized equations of parabolic type and based on the ideas expressed in [5, 6].

Consequently, the purpose of research is a synthesis of minimax boundary distributed and point regulators of the observed variables, determining number and optimal location of point regulators.

The main material research

To formulate correct mathematical formulation of the problem, we enter the following notation: *Q* c *Rn* - limited open area with piecewise smooth boundary Г; *QT* ={(x, t): *x* є Q, 0 < *t* < *T*}, *ST* = {(x, t): *x* є Г, 0 < *t* < *T*}, де *T* ; *(•,•)*((•,•) Г) – scalar multiplication in Hilbert space L2(Q) (L2(г)); (•,•) - Euclidean scalar multiplication; " *T* " – operation of transposing; " * " – conjugation operation of operators; *Hk*(Q), *Hk'k(QT*) – Sobolewski spaces [7];

$L_2(V,R^N)=\left\{f : f \quad [f_1,f_2,...,f_N]^T ,\ \int\limits_V \| f(x) \|_{R^N}^2\, dx < \infty\right\}$;r/ $L\left(V,H\right)$ – space of continuous linear operators acting on a Hilbert space V in the Hilbert space H; *A(t)* – elliptic operator of the second order of the form:

$$A(t)=\sum_{i,j=1}^{n}\frac{\partial}{\partial x_i}\left(a_{i,j}(x,t)\frac{\partial}{\partial x_j}\right)-a_0(x,t), \tag{1}$$

where $a_0(x,t)$, $a_{i,j}(x,t)$ – functions that are defined in the cylinder and satisfy the following conditions: $a_0 \in C(Q_T)$, $a_{i,j} \in C^1(Q_T)$, $a_0 \geq 0$ almost everywhere in Q_T, $\sum_{i,j=1}^{n} a_{i,j}(x,t)\xi_i\xi_j \geq \alpha \sum_{i=1}^{n}\xi_i^2$, $\alpha > 0$ $\forall \xi_i \in R^1$ almost everywhere in Q_T; $\partial/\partial \nu_A$ –

corresponding to operator $A(t)$ of conormal derivative operator $\dfrac{\partial \varphi}{\partial \nu_A} = \sum_{i,j=1}^{n} a_{i,j}(x,t) \dfrac{\partial \varphi}{\partial x_j} \cos(\vec{n}, x_i)$, where $\cos(\vec{n}, x_i)$ – i – directional cosine of outer normal $\vec{n}$ to the border Γ of the area Ω.

Let the state of the system described by the function $\varphi(x,t)$, which satisfies the equation

$$\int_0^T \langle \varphi(t), W^*(t)\eta(t) \rangle dt = \int_0^T b(t; u(t), \eta(t)) dt + m(f, \eta(0)) \quad \forall \eta(t) \in \Phi_T, \tag{2}$$

where $W(t) = \partial/\partial t - A(t)$; $m(f, \eta(0))$, $b(t; u(t), \eta(t))$ – continuous bilinear forms;

Φ_T – space of "test" functions $\eta(t)$ by the type of $\Phi_T = \left\{ \eta :\ \eta \in H^{2,1}(Q_T),\ \eta|_{S_T} \quad 0;\ \eta(x,T) \quad 0,\ x \in \Omega \right\}$; $u \in U$ – management functions ($U = L_2(S_T)$ – for distributed control limit; $U = L_2(S_T; R^N)$ – for management concentrated; $f \in L_2(\Omega)$ – unknown functions which belong area

$$S_f = \{ f : f \in L_2(\Omega),\ h(f,f) \le 1 \}, \tag{3}$$

where $h(f,f)$ - symmetric positively defined quadratic form.

Note that by made assumptions for each management $u \in U$ solution of equation (2) exists and it is only in space $L_2(Q_T)$ [8].

Suppose that at some realization external disturbances $f \in S_f$ occur following dimensions of the system (2)

$$z_i(t) = l_i(t; \varphi(t)) = \langle l_i(t), \varphi(t) \rangle,\ \ z_i(t) \in L_2(0,T),\ \ i = 1,2,\dots,k, \tag{4}$$

where $l_i(t) \in L_2(Q_T)$, $i = 1,2,\dots,k$ – linearly independent functions.

The task is to find control $u(t)$ in a linear feedback from the observed signals $z(t) = [z_1(t), z_2(t), \dots, z_k(t)]^T$ i.e. in the form

$$u(t) = R(t) z(t), \quad R(t) \subset L\left(L_2(0,T; R^k), U \right), \tag{5}$$

which minimizes the following functional in the equation solution (2)

$$I(u) = \sup_{f \in S_f} \left[q(\varphi(T), \varphi(T)) + \int_0^T \left(p(t; \varphi(t), \varphi(t)) + d(t; u(t), u(t)) \right) dt \right]. \tag{6}$$

There are presented the following notation as $q(\varphi(T), \varphi(T))$, $p(t; \varphi(t), \varphi(t))$ – symmetrical integrally defined quadratic forms, $d(t; u(t), u(t))$ – symmetric positively defined quadratic form.

Formulated problem will be called optimization task of minimax management, and function $u(t)\in U$ that delivers infimum of functional (6) – minimax optimal control. Denote by $M, B(t), H, Q, P(t), D(t)$ operators, generated by bilinear and quadratic forms $m(f,\eta)$, $b(t;u(t),\eta(t))$, $h(f,f)$, $q(\varphi(T),\varphi(T))$, $p(t;\varphi(t),\varphi(t))$, $d(t;u(t),u(t))$ accordingly.

The main results of this work represented as the following theorem

Theorem 1.

A) Solution of minimax control problem (2), (5), (6) and minimax optimal management, that satisfy the necessary conditions of optimality is determined by the correlation (5) ,where operator of feedback $R(t)$ satisfies the equation:

$$\int_0^T \Big(d(t;R(t)L(t)\psi(t),\Theta(t)L(t)\psi(t)) + b(t;\Theta(t)L(t)\psi(t),K(t)\psi(t)) \Big) dt = 0 \qquad (7)$$

$$\forall\ \Theta(t)\in L\Big(L_2(Q_T;R^k),U\Big),$$

where $L(t)\in L\Big(L_2(Q_T),L_2(Q_T,R^k)\Big)$ – operator of type $L(t)=\langle l(t),\cdot\rangle$, which operates by the rule $L(t)\eta(t)=\langle l(t),\cdot\rangle\eta(t)=\langle l(t),\eta(t)\rangle$; $\psi(t)$ – the solution of equation.

$$\int_0^T \Big\langle \psi(t),W^*(t)\eta(t)\Big\rangle dt = \int_0^T b(t;R(t)L(t)\psi(t),\eta(t))dt + m(l_{\max}(V),\eta(0)) \quad \forall\ \eta(t)\in\Phi_T\,, \ (8)$$

$l_{\max}(V)\in L_2(\Omega)$ – own function that corresponds to the maximum eigenvalues $\lambda_{\max}(V)$ of operator $V=H^{-1}M^*K(0)M$; $K(t)$ – self-adjoint positively defined operator that satisfies the equation:

$$\int_0^T \Big\langle K(t)\eta(t),W(t)\zeta(t)\Big\rangle dt + \int_0^T \Big\langle K(t)\zeta(t),W(t)\eta(t)\Big\rangle dt \quad \int_0^T b(t;R(t)L(t)\zeta(t),K(t)\eta(t))dt +$$

$$+\int_0^T b(t;R(t)L(t)\eta(t),K(t)\zeta(t))dt + \int_0^T d(t;R(t)L(t)\eta(t),R(t)L(t)\zeta(t))dt +$$

$$+\int_0^T p(t;\eta(t),\zeta(t))dt + q(\eta(T),\zeta(T)) \qquad \forall\ \eta(t),\zeta(t)\in\Phi_0, \tag{9}$$

where $\Phi_0 = \left\{\eta : \eta \in H^{2,1}(Q_T),\ \eta\big|_{S_T}\ \ 0,\ \eta(x,0)\ \ 0,\ x\in\Omega\right\}$.

For all that the value of the functional on the optimal management can be represented in the type:

$$I(u) = \lambda_{\max}(V) = \lambda_{\max}(H^{-1}M^{*}K(0)M). \tag{10}$$

B) One of solving of the optimization problem (2), (5), (6), that satisfies the necessary conditions for optimality is determined by the correlation:

$$u_0(t) = R_0(t)z(t), \qquad R_0(t) = -D^{-1}(t)B^{*}(t)K(t)l^{T}(t)\left\langle l(t),l^{T}(t)\right\rangle^{-1}, \tag{11}$$

where $l(t) = \left[l_1(t),l_2(t),\ldots,l_k(t)\right]^T$, $\left\langle l(t),l^T(t)\right\rangle = \left\{\left\langle l_i(t),l_j(t)\right\rangle\right\}_{i,j=1}^{k}$ – matrix of Gramm [8], $K(t)$ – the solution of the following equation:

$$\int_0^T \left\langle K(t)\eta(t),W(t)\zeta(t)\right\rangle dt + \int_0^T \left\langle K(t)\zeta(t),W(t)\eta(t)\right\rangle dt - q(\eta(T),\zeta(T)) =$$

$$= -\int_0^T \left\langle B(t)D^{-1}(t)B^{*}(t)K(t)\eta(t),K(t)\zeta(t)\right\rangle dt + \int_0^T p(t;\eta,\zeta)dt \quad \forall\ \eta(t),\zeta(t)\in\Phi_0. \tag{12}$$

The value of criterion on minimax management also is defined by the formula (10), where $K(t)$ in this case – is the solution of the equation (12). The proof of the theorem is not given here. We only note that it is based on the ideas of work and involves the use of Rayleigh roughness and methods of perturbation theory [10].

Note 1. If disturbances affect the system is not only at the initial time, but also at affect the all-time of regulation, then the problem discussed above (which was considered above) has no solution.

Note 2. Suppose that bilinear forms $b(t;u,\eta)$ and $d(t;u,u)$ has a type

$$b(t;u,\eta) = \begin{cases} -\int_\Gamma\int_\Gamma B(x,y,t)u(y,t)\dfrac{\partial\eta(x,t)}{\partial\nu_{A_x^*}}dxdy, & \text{if} \quad U = L_2(S_T), \\ -\sum_{i=1}^{N} u_i(t)\int_\Gamma b_i(x,t)\dfrac{\partial\eta(x,t)}{\partial\nu_{A_x^*}}dx, & \text{if} \quad U = L_2(S_T;R^N), \end{cases} \tag{13}$$

$$d(t;u,u) = \begin{cases} -\int_\Gamma\int_\Gamma D(x,y,t)u(x,t)u(y,t)dxdy, & \text{if} \quad U = L_2(S_T), \\ \left(D(t)u(t),u(t)\right), & \text{if} \quad U = L_2(S_T;R^N), \end{cases} \tag{14}$$

where $B(x,y,t)$, $D(x,y,t) \in L_2(0,T;L_2(\Gamma)\times L_2(\Gamma))$ moreover $D(x,y,t)$ – symmetric positively defined function; $D(t)-$ symmetrical positively defined matrix which elements belong to the space $L_2(0,T)$; $b_i(x,t) \in L_2(S_T)$; $u(t)=\left[u_1(t),u_2(t),...,u_N(t)\right]^T$, $u_i(t)\in L_2(0,T)$.

Then, using formally second Green's formula, the equation (2) can be interpreted as the boundary value problem of Dirichlet with the boundary control

$$\begin{cases} \dfrac{\partial \varphi(t)}{\partial t} = A(t)\varphi(t) \quad \text{in the area} \quad Q_T, \\ \varphi(0) = Mf \quad \text{in the area} \quad \Omega, \quad \varphi(t) = B(t)u(t) \quad \text{in the area} \quad S_T, \end{cases} \tag{15}$$

$$B(t)u(t) = \begin{cases} \int\limits_\Gamma B(x,y,t)u(y,t)dy, & \text{if} \quad U = L_2(S_T), \\ \sum\limits_{i=1}^N b_i(x,t)u_i(t), & \text{if} \quad U = L_2(S_T;R^N). \end{cases} \tag{16}$$

Note 3. If bilinear forms $b(t;u,\eta)$ and $d(t;u,u)$ satisfy correlation (13), (14), so the core $K(x,y,t)$ of an operator $K(t)$ which is the solution of equation (12), formally satisfy the following integral-differential equation of Riccati type

$$\begin{aligned} \frac{\partial K(x,y,t)}{\partial t} &= -A_x^*(t)K(x,y,t) - A_y^*(t)K(x,y,t) + \\ &+ \int\limits_\Gamma\int\limits_\Gamma \frac{\partial K(x,\xi,t)}{\partial \nu_{A_\xi^*}} G(\xi,\eta,t) \frac{\partial K(y,\eta,t)}{\partial \nu_{A_\eta^*}} d\xi d\eta - P(x,y,t) \end{aligned} \tag{17}$$

with initial and boundary conditions of the type

$$\begin{cases} K(x,y,T) = Q(x,y), & (x,y)\in \Omega_x \times \Omega_y; \\ K(x,y,t) = 0, & (x,y,t) \in \Gamma_x \times \Omega_y \times (0,T); \\ K(x,y,t) = 0, & (x,y,t) \in \Omega_x \times \Gamma_y \times (0,T), \end{cases}$$

where

$$G(x,y,t)=\begin{cases}\int\limits_{\Gamma}\int\limits_{\Gamma} B(x,\xi,t)D^{-1}(\xi,\eta,t)B(y,\eta,t)d\xi d\eta, & \text{if} \quad U=L_2(S_T),\\ B^T(x,t)D^{-1}(t)B(y,t), & \text{if} \quad U=L_2(S_T;R^N),\end{cases} \tag{18}$$

$Q(x,y)$, $P(x,y,t)$, $D^{-1}(x,y,t)$ – cores of operators Q, $P(t)$ and $D^{-1}(t)$ accordingly; $B(x,t)=\left[b_1(x,t),b_2(x,t),...,b_N(x,t)\right]^T$; indices operators $A(t)$, $\partial/\partial\nu_A$ indicate at which variable these operators act.

Denote by $u^k(t)$ optimal minimax control (11), obtained at k measurements (4) and consider the problem of construction recurrent algorithm of definition optimal control of initial optimization problem relatively the change in the number of observations k. Solution of this problem is given by the following theorem.

Theorem 2. The optimal minimax management $u^k(t)$ is determined by the following recurrent procedure

$$\begin{cases} u^k(t)=u^{k-1}(t)+h_{k-1}^{-1}(t)F(t)V_{k-1}(t)l_k(t)\left(z_k(t)-l_k(t;F^+(t)u^{k-1}(t))\right),\\ u^0(t)=0, \qquad k=1,2,..., \end{cases} \tag{19}$$

where $F(t)=-D^{-1}(t)B^*(t)K(t)$, $h_{k-1}(t)=\langle l_k(t),V_{k-1}(t)l_k(t)\rangle$, "$+$" – an operation of pseudoinversion operators [10], $V_k(t)\in L\left(L_2(Q_T),L_2(Q_T)\right)$ – selfadjoint operator that satisfy the following recurrent equation

$$\begin{cases} V_k(t)=V_{k-1}(t)-h_{k-1}^{-1}(t)V_{k-1}(t)l_k(t)\langle V_{k-1}(t)l_k(t),\cdot\rangle,\\ V_0(t)=E, \end{cases} \tag{20}$$

where E – the identity operator.

The proof of the theorem is carried out by using formulas of inversion block matrix operators [7,10].

Note 4. If $l_i(t)$, $i=1,2,...,k$ – linearly independent orthonormal in the space $L_2(\Omega)$ system of functions, namely $\langle l_i(t),l_i(j)\rangle=\delta_{ij}$, where δ_{ij} – Kronecker symbol, then the optimal control satisfies the following recursive equation

$$\begin{cases} u^k(t)=u^{k-1}(t)+F(t)l_k(t)z_k(t), \qquad k=1,2,3,..., \\ u^0(t)=0. \end{cases} \tag{21}$$

Whereas the effectiveness of management is determined by the quality criterion for this control, then it is considered in more detail the value of the functional (6) on optimal control (11). According to Theorem 1 it is determined by the following expression

$$I(u_0)=\lambda_{\max}(H^{-1}M^*K(0)M), \tag{22}$$

where the operator $K(t)$ satisfy the equation (12). It is obviously, that to calculate $I(u_0)$ in general is quite difficult as for this purpose we must solve two difficult problems. The first problem is solving Ricatti equation and the second is determination of the maximum eigenvalue of infinite measurable operator. Therefore, let us stop on some partial cases where the value is calculated rather simply.

1.Consider the case of a distributed control limit, namely a case when bilinear form

$b(t;u(t),\eta(t))$ is given by the formula (13) on condition $U=L_2(S_T)$, in which $B(x,y,t)=b(x)\delta(x-y)$, where $\delta(x-y)$ – Dirac delta function. Bilinear and quadratic forms $m(f,\eta)$, $h(f,f)$, $q(\varphi(T),\varphi(T))$, $p(t;\varphi,\varphi)$, $d(t;u,u)$ define by the following way

$$m(f,\eta)=\int_\Omega m(x)f(x)\eta(x)dx,\ \ h(f,f)=\int_\Omega h(x)f^2(x)dx,\ \ d(t;\varphi,\varphi)=\int_\Gamma d(x)u^2(x,t)dx,$$

$$q(\varphi(T),\varphi(T))=\int_\Omega q(x)\varphi^2(x,T)dx,\quad p(t;\varphi,\varphi)=\int_\Omega p(x)\varphi^2(x,t)dx,$$

where $q(x)\geq 0$, $p(x)\geq 0$, $h(x)>0$, $d(x)>0$.

Then in the assumption that $A(t)$ – self-adjoint independent of time t operator, namely $A(t) = A = A^*$ can be shown that the value of the functional is equal to $I(u_0) = \max\limits_{1 \le i < \infty} \frac{m_i^2}{h_i} k_i$, where

$$k_i = \alpha_i^{-1} \left[\mu_i \frac{p_i \alpha_i - \lambda_i + \text{th}(\mu_i T)}{(p_i \alpha_i - \lambda_i)\,\text{th}(\mu_i T) + \mu_i} + \lambda_i \right], \quad \mu_i = \sqrt{\lambda_i^2 + \alpha_i q_i}\,,$$

$$\begin{pmatrix} m_i \\ h_i \\ q_i \\ p_i \end{pmatrix} = \int_\Omega \begin{pmatrix} m(x) \\ h(x) \\ q(x) \\ p(x) \end{pmatrix} \omega_i^2(x) dx, \quad \alpha_i = \int_\Gamma \frac{b^2(x)}{d(x)} \left(\frac{\partial \omega_i(x)}{\partial \nu_{A_x}} \right)^2 dx.$$

In the last formulas marked: $\text{th}(\cdot)$ – hyperbolic tangent; λ_i and $\omega_i(x) \in L_2(\Omega)$ – eigenvalues and corresponding orthonormal in the space $L_2(\Omega)$ the eigenfunctions of operator that satisfy the equation

$$\begin{cases} \langle \omega_i, A\eta \rangle = \lambda_i \langle \omega_i, \eta \rangle \qquad \forall\ \eta \in H^2(\Omega) \cap H_0^1(\Omega), \\ \omega_i(x) = 0,\ x \in \Gamma; \quad \lambda_i \to -\infty \ \text{при} \ i \to \infty. \end{cases} \tag{23}$$

2. Now let bilinear form $b(t; u(t), \eta(t))$ is determined by the ratio (13), in which the set of admissible controls $U = L_2(S_T; R^N)$, namely lets consider a case of limit focused controls. Relatively quadratic forms $q(\varphi(T), \varphi(T))$, $p(t; \varphi(t), \varphi(t))$, $d(t; u(t), u(t))$ suppose that

$$q(\varphi(T), \varphi(T)) = \langle q, \varphi(T) \rangle^2, \quad p(t; \varphi(t), \varphi(t)) = 0, \quad d(t; u(t), u(t)) = \sum_{i=1}^N d_i(t) u_i^2(t),$$

where $q \in L_2(\Omega)$; $d_i(t) \in L_2(0,T)$, $d_i(t) > 0$.

Then, using the results of the work [6,9], it can be shown that the functional value (22) equal to

$$I(u_0) = \lambda_{\max}\left(\nu(0) H^{-1} M^* r(0) \langle M^* r(0), \cdot \rangle \right) = \nu(0) \langle H^{-1} M^* r(0), M^* r(0) \rangle, \tag{24}$$

where

$$\nu(t) = \left(1 + \int_t^T \alpha(\tau) d\tau \right)^{-1}, \ \alpha(t) = \sum_{i=1}^N \frac{1}{d_i(t)} \left\langle b_i(t), \frac{\partial r(t)}{\partial \nu_A} \right\rangle_\Gamma^2, \ r(t) = \sum_{i=1}^\infty e^{\lambda_i (T-t)} \langle q, \omega_i \rangle \omega_i\,, \tag{25}$$

λ_i and ω_i – eigenvalues and the corresponding eigenfunctions of the operator A that satisfy the equation (23).

Consider now the point boundary control $u(t) \in U = L_2(S_T; R^N)$. For this in (13) put $b_i(x,t) = \delta(x - x_i)$, $x_i \in \Gamma$, $i = 1, 2, ..., N$. Then the equation (2) describes the system with point boundary controls. We note that this range of functions $b_i(x,t)$ allowed under certain restrictions on the dimension of space $\Omega \subset R^n$ and on condition a higher smoothness of "test" functions $\eta(t)$ in (2). In particular it is possible if you put $n \le 3$ and require that functions $\eta(t)$ belong not to the space $H^{2,1}(Q_T)$, as it was supposed above, but belong to more sleek space of functions $H^{4,1}(Q_T)$.

We introduce the following definition

$$J_N(x_1, x_2, ..., x_N) = \inf_{u \in L_2(S_T; R^N)} \sup_{f \in S_f} \left[\langle q, \varphi(T) \rangle^2 + \int_0^T \sum_{i=1}^N d_i(t) u_i^2(t) dt \right] \tag{26}$$

and consider the problem of determining a such number of regulators N in the form of feedback (5) and of their optimum location $\left(x_1^0, x_2^0, ..., x_N^0\right)$, $x_i^0 \in \Gamma$ at which the condition is executed

$$J_N(x_1^0, x_2^0, ..., x_N^0) = \inf_{x_i \in \Gamma,\, i=1,2,...,N} J_N(x_1, x_2, ..., x_N) < \varepsilon, \tag{27}$$

where $\varepsilon > 0$ – some pre-set threshold value.

Using Theorem 1 and the ratio (24)(25), there can prove fairness of the following theorem.

Theorem 3. The number of regulators by which the inequality is done satisfies condition $N \ge N_0$, where

$$N_0 = \left[\frac{\Delta - \varepsilon}{\varepsilon \beta \gamma} \right] + 1, \tag{28}$$

$$\Delta = \left\langle H^{-1} M^* r(0), M^* r(0) \right\rangle, \quad \beta = \sup_{x \in \Gamma} \int_0^T \left(\frac{\partial r(x,t)}{\partial \nu_{A_x}} \right)^2 dt, \quad \gamma = \min_i \left(\max_{t \in (0,T)} d_i(t) \right)^{-1}, \tag{29}$$

$[\cdot]$ – an entire part of number. All regulators should be concentrated at one point which is defined as follows

$$x_0 = \arg\sup_{x \in \Gamma} \int_0^T \left(\frac{\partial r(x,t)}{\partial \nu_{A_x}} \right)^2 dt. \tag{30}$$

Note 5. The last theorem can be formulated also the following way. In order to get performed inequality (27) it is need only one point regulator at the point (30) of total intensity $\sum_{i=1}^{N} u_i(t)$, $N \geq N_0$, where

$$u_i(t) = R_i(t)z(t), \quad R_i(t) = -\frac{\nu(t)}{d_i(t)} \frac{\partial r(x,t)}{\partial \nu_{A_x}}\bigg|_{x=x_0} \int_{\Omega} r(y,t) l^T(y,t) dy \left\langle l(t), l^T(t) \right\rangle^{-1}$$

functions $\nu(t)$, $r(x,t)$ determined by formulas (25), $z(t)$ – Observations type of (4), and N_0 satisfying ratio (28).

Note 6. If $d_i(t) \equiv d \quad \cancel{const} > 0$, $i = 1,2,3,...$, then N_0 – minimal number of regulators that satisfy inequalities (27).

CONCLUSIONS

The solution of several problems of synthesis of optimal control of distributed systems of parabolic type, which operate under conditions of uncertainty, is proposed. In addition, the solution of the problem of optimal location of the point limiting regulators and determination of their number is given.

Acknowledgment

The work was done and financially supported at the Department of automation and intelligent control systems, computer systems automation faculty of the National University of Food Technologies in. Kiev, Ukraine.

References

1. *Кириченко, Н.Ф.* Минимаксное управление и оценивание в динамических системах /Kirichenko, N.F. Minimax Control and Estimation in Dynamic Systems /Автоматика/ Automation /. – 1982. – №1. – P. 32 – 39.
2. *Наконечный, А.Г.* Минимаксные оценки параметров / Nakonechny, A.G. Minimax estimation of parameters / Вычислительная и прикладная математика / Computational and Applied Mathematics /. – 1979. – Vol.39. – P. 17 – 24.
3. *Наконечный, А.Г.* Мінімаксне оцінювання функціоналів від розв'язку крайових задач для параболічних рівнянь при точкових спостереженнях / Nakonechniy, A.G. Mynymaks evaluating functional solution of boundary value problems for parabolic equations with point observations/ Вісник Київського університету / Bulletin of Kiev University /, Серія фізико-математичні науки / Series of Physical-Mathematical Sciences /. – 2001. – Vol.1. – P. 191–196.

4. *Наконечний, О.Г.* Оцінювання параметрів в умовах невизначеності / Nakonechny, O.H. Estimation of parameters in uncertainty // Наукові записки КНУ ім. Т.Г. Шевченка / Scientific notes KNU. TH Shevchenko /. – 2004. – Vol.7. – P. 102 – 112.
5. *Лобок, А.П.* Минимаксные регуляторы в системах с распределенными параметрами / Loboc, A.P. Minimax regulators in systems with distributed parameters / Вестник Киевского университета. Моделирование и оптимизация сложных систем / Bulletin of the University of Kiev. Simulation and optimization of complex systems /. – 1983. – Vol.2. – С. 62 – 67.
6. *Лобок, О.П. та інші.* Синтез оптимального мінімаксного керування лінійними багатовимірними об'єктами за умови неточного і неповного їх вимірювання / Loboc, A.P. and others. Synthesis minimax optimal control of linear multidimensional objects provided inaccurate and incomplete to measure / Збірник наукових праць КНТУ.: Кіровоград. / Proceedings of KNTU .: Kirovograd./ – 2013. – Vol 26. – P. 124 – 132.
7. *Люстерник, Л.А.* Элементы функционального анализа / Lyusternik, L.A. Elements of functional analysis/ – М.: Наука /Science /, 1979. – 520 p.
8. *Наймарк, М.А.* Линейные дифференциальные операторы / Naimark, M.A. Linear differential operators/ – М.: Наука / Science /, 1969. – 526 с.
9. *Лобок, О.П.* Мінімаксне управління лінійними багатовимірними об'єктами зі стаціонарними зовнішніми збуреннями / Loboc, A.P. and others. Minimax control linear multidimensional objects from stationary external perturbations / Наукові праці Національного університету харчових технологій. / Proceedings of the National University of Food Technologies./ – Київ: НУХТ / Kyiv: NUFT /. – 2013. – № 46. – P. 43 – 51.
10. *Като, Т.* Теория возмущений линейных операторов / Kato, T. Perturbation theory for linear operators / – М.: Мир / Peace /, 1972. – 739 p.

А.П Лобок, доц., Канд. техн. наук, Б.М. Гончаренко, проф., Докт. техн. наук

Національний університет харчових технологій, Київ, Україна

Л.Г. Віхрова, проф., Канд. техн. наук.

Кіровоградський національний технічний університет, Кропивницький, Україна

Оптимальне керування лінійними динамічними розподіленими системами в умовах невизначеності

У статті розглянуті задачі синтезу оптимального керування системами, що функціонують в умовах невизначеної інформації й описуються узагальненими рівняннями в частинних похідних параболічного типу. Керування має вигляд зворотного зв'язку від спостережуваних вимірів, для реалізації якого необхідно розв'язати інтегро-диференціальне рівняння типу Ріккаті. Окремо побудовані розподілені та зосереджені граничні регулятори, а також наведено рекурентний алгоритм визначення оптимального керування стосовно зміни числа спостережень. Розроблено алгоритм визначення необхідної кількості точкових регуляторів та їх оптимальне розташування на границі області, при яких критерій якості не перевищує заданого порогового значення.

Ключові слова: мінімаксне керування, точкові граничні регулятори, соболевські простори, нерівність Релея, білінійна форма.