

## Experimental and theoretical study of ice formation on vertical cooled pipes

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### Abstract

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**Introduction.** The use of cold accumulators based on the principle of ice build up on the cooled surfaces during off-peak periods and ice melting during on-peak periods is an effective method of electricity bills reduction.

**Materials and methods.** Dynamics of ice accumulation on the surface at different  $\Delta t$  (refrigerant evaporation temperature and the temperature of water that overflows surface) has been studied. Series of experiments have been carried out with 2 refrigerants (R12 and R22). The temperatures of water and that of refrigerant evaporation have varied within the intervals  $+1,5\div+4,5^{\circ}\text{C}$ ;  $-10\div-20^{\circ}\text{C}$ , respectively. The mass flow rate and the velocity of water within the experimental sections were kept constant during a whole series of experiments. The ice-layer thickness was measured by means of optical method. The instantaneous images of the experimental pipe with the ice layer were processed with the graphic processing software.

**Results and discussion.** Since within comparatively short periods of on-peak demand a noticeable amount of thermal energy related to ice melting is to be released, it becomes clear that the sizing of ice accumulators based on a simple balance calculations is actual, but also the determination of time periods of ice accumulation and ice thawing becomes critical. The derivation of a simple differential equation of ice formation on the vertical cooled pipe, which then is used as a core for semi-empirical correlation of experimental data obtained on a special experimental unit is presented. This approach allowed elimination of a number of regime parameters used in the differential equation, which could not be determined directly. A correction factor which correlates the numerical solution of the differential equation and experimental data has been obtained. The asymptotic values of ice thickness varied within 4...16 mm at different  $\Delta t$  and water flow rates. Thus generalized correlations will allow to determine an optimal amount of ice to be stored in the cold accumulator and eventually to significantly reduce energy consumption by approximately 10-15 %. The process differential equation has been derived with the following suggestions: the problem is one dimensional, the ice is growing in radial direction. Given were: the heat transfer coefficient from water to ice surface and from copper pipe to evaporating refrigerant. The equation has been derived with taking into account the infinitesimal increment of ice build at a time interval  $\Delta\tau$  and corresponding heat balance.

**Conclusions.** Differential equation can be used for the determination of time period necessary for the accumulation of a given amount of water ice.

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## Introduction

Electrical motors of refrigeration compressors are the biggest energy consumers at food industry. Dairy industry is characterized by a significant refrigeration capacity for water cooling and extremely uneven energy consumption charts. Ice water is being used in raw milk pasteurization which usually happens right after milk reception at dairy plant. This peak cooling load usually is accompanied by a serious growth of cooling load due to the increased sensitive head influx through the building envelope, increased input of product. Unfortunately, the factory on-peak electricity consumption coincides with the grid on-peak energy demand, and thus, energy consumed during peak periods has to be paid at the highest tariff.

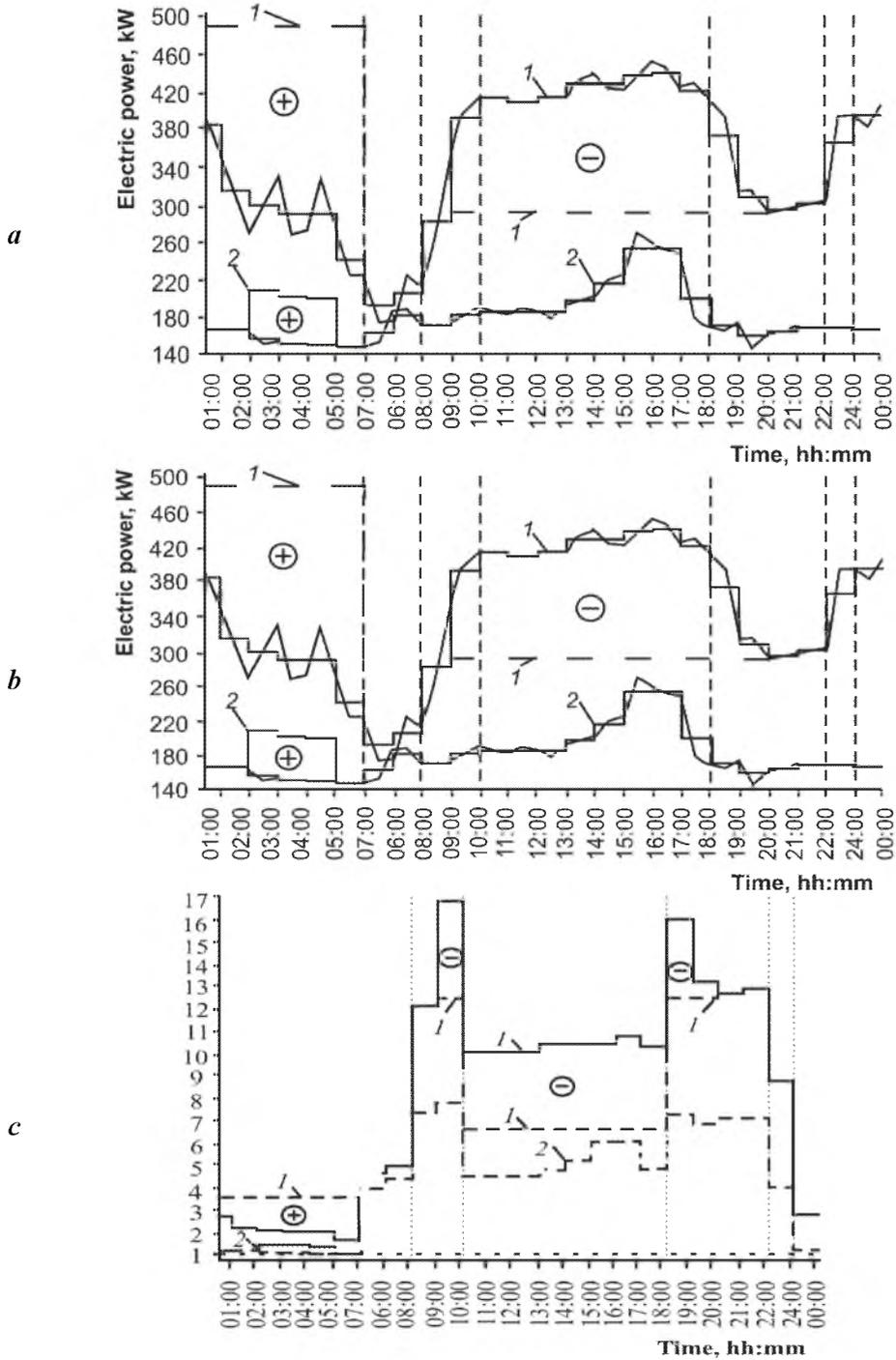
Modern office building and malls are equipped with powerful air-conditioning plants (Installed power up to 12-20 MW). Growth in energy consumption takes place at mid days and coincides with mild peaks or on-peak period of grid power demand. A typical energy demand chart registered at a Ukrainian dairy plant (a) along with the modified energy consumption chart (b) and hourly money flow chart (c) are shown in Fig. 1.

The original chart of power demand can be modified by reducing the morning on-peak load between 09<sup>00</sup> and 10<sup>00</sup> then by lowering load during the semi-peak period and eventually cutting the on-peak within 18<sup>00</sup> – 20<sup>00</sup> period. This cooling load can be shifted to the night period between 00<sup>00</sup> and 06<sup>00</sup>. The redistribution of loads is shown by the red zone marked with “+” sign on the lines 1 and 2 which mean a load increase at night. The reduction of load is shown by greenish zones marked with “-” sign on the line 1.

The result of the modification can be seen on Fig.1 (c) by dotted lines beneath the solid lines 1 and 2 showing money flow for the original case and beneath the dotted lines after loads shift.

A total of 5,1% reduction in daily electricity bills thus is easily attainable. An extended analysis of the possible options of power demand shifts is given in [3, 4].

The shifting of energy consumption can be achieved by using cold storages in which cold is accumulated by ice formation during night lowest grid demand periods and release of the stored cold during on-peak demand periods by melting ice and partially unloading refrigeration compressors [5].



**Fig. 1. Power demand and money flow chart**

- a – Original power demand for refrigeration compressors on lines 1 and 2;
- b – Modified power demand by shifting time load of compressors;
- c – Hourly expenses for consumed electricity, relative units.

### Mathematical model

A cylindrical problem is used for the ice build-up model formulation. The following simplifications and suggestions have been taken: one-dimensional case, constant physical parameters of water and refrigerant, no super cooling of water, no water density anomaly.

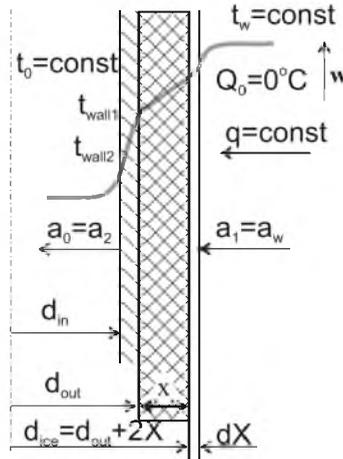


Fig. 2. Ice build-up model

The heat released during  $d\tau$  due to the water friction on the water-ice interface is determined by:

$$q_{fr} = \frac{h_w \cdot \omega^2 \cdot \pi \cdot (d_{out} + 2 \cdot x) \cdot \left(2 + \frac{1}{Pr^w}\right)}{3 \cdot C^w \cdot p} \cdot d\tau, \quad (1)$$

as it is recommended in [6].

The amount of heat to be absorbed in order to build-up an ice layer of  $2x$  thickness equals:

$$q_{ice} = \frac{\pi \cdot \rho \cdot 2 \cdot dx}{4} \cdot (2 \cdot d_{out} + 4 \cdot x + 2 \cdot dx) = \pi \cdot \rho \cdot dx \cdot (d_{out} + 2 \cdot x) \quad (2)$$

The amount heat that has to be transferred through the multi layer cylindrical wall to the evaporating refrigerant inside the tube is:

$$q_0 = \frac{\pi \cdot (\theta_0 - t_0)}{\frac{1}{2 \cdot k_{ice}} \cdot \ln \frac{d_{out} + 2 \cdot x}{d_{out}} + \frac{1}{2 \cdot k_m} \cdot \ln \frac{d_{out}}{d_{in}} + \frac{1}{h_r \cdot d_{in}}} \cdot d\tau \quad (3)$$

The heat gain due to the heat transfer from water to the ice surface may be described as:

$$q_w = \pi \cdot d_{ice} \cdot h_w \cdot (t_w - \theta_0) \cdot d\tau = \pi \cdot (d_{out} + 2 \cdot x) \cdot h_w \cdot (t_w - \theta_0) \cdot d\tau \quad (4)$$

It is clear that the amount of heat transferred to the evaporating refrigerant (3) equalizes the heat gained from the out flowing water (4), ice fusion (2) and interface friction (1), thus giving a differential equation if ice build-up time rate:

$$\frac{dx}{d\tau} = \frac{\left\{ \frac{\theta_0 - t_0}{\frac{1}{2 \cdot k_{ice}} \cdot \ln \frac{d_{out} + 2 \cdot x}{d_{out}} + \frac{1}{2 \cdot k_m} \cdot \ln \frac{d_{out}}{d_{in}} + \frac{1}{h_r \cdot d_{in}}} \right\} \cdot \left[ -h_w \cdot (d_{out} + 2 \cdot x) \cdot (t_w - \theta_0) + \frac{\omega^2 \cdot \left( 2 + \frac{1}{Pr^w} \right)}{3 \cdot C_p^w} \right]}{\rho \cdot H \cdot (d_{out} + 2 \cdot x)}, \quad (5)$$

with the boundary condition :  $\tau = 0, x = 0$ .

Heat transfer coefficients  $h_w$  and  $h_r$  were calculated by the empirical correlations [7, 11].

$$Nu = Nu_\infty + f\left(\frac{d_i}{d_0}\right) \cdot \frac{0.19 \cdot \left[ Pe \cdot \left( \frac{d_h}{L} \right) \right]^{0.8}}{1 + 0.117 \cdot \left[ Pe \cdot \left( \frac{d_h}{L} \right) \right]^{0.467}},$$

$$Nu_\infty = 3.66 + 1.2 \cdot \left( \frac{d_i}{d_0} \right)^{-0.8},$$

$$f\left(\frac{d_i}{d_0}\right) = 1 + 0.14 \cdot \left( \frac{d_i}{d_0} \right)^{0.5} \quad \text{and} \quad h_r = 3.1 \cdot p^{0.25} \cdot q^{\frac{2}{3}}. \quad (6)$$

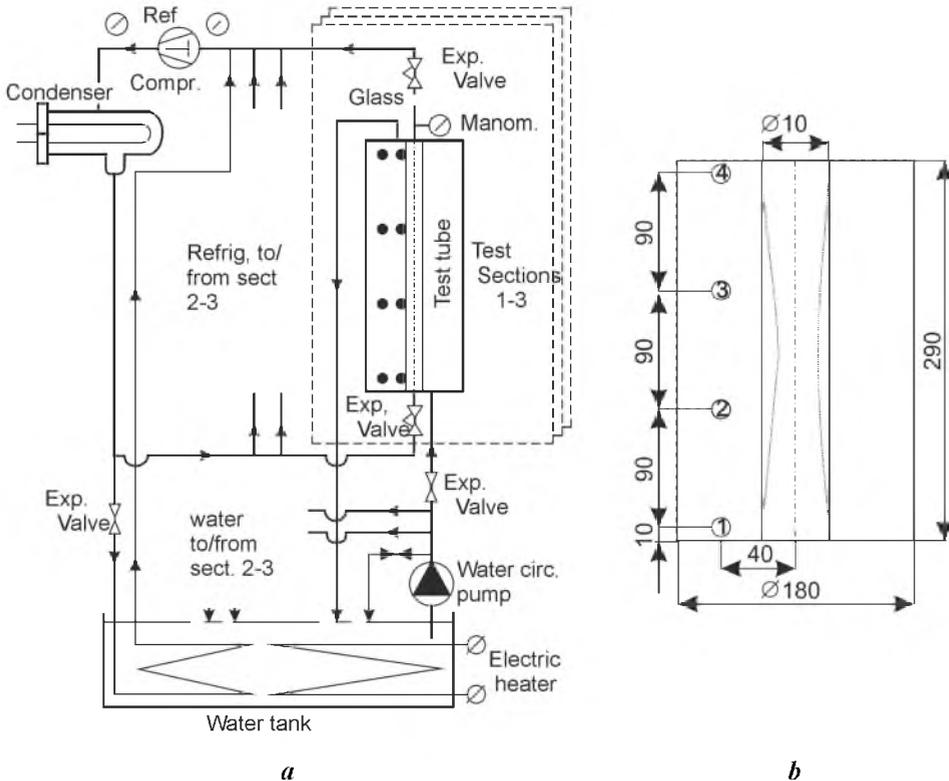
From the analysis of equations (1-6) it can be seen that the coefficient  $h_r$  depends on heat flux  $q$  (6), at the same time the value of  $q$  is determined by the thickness of ice layer which grows gradually,  $x$  in equation (3 and 5).

The equation (5) can be solved numerically on the time intervals beginning with  $\tau=0$ . The thickness of ice  $x_{i-1}$  achieved on the previous interval should be used as a boundary condition for the next time interval. Similarly, a new value of  $h_{ri}$  which has to be introduced in (5) is to be determined by solving (6) with a new value of  $q$ .

The piecewise solution so obtained has to be compared with the data obtained experimentally.

### Experimental rig

A lay-out of the experimental rig designed and constructed for the determination of ice formation time rate is shown in Fig.3



**Fig. 3. Experimental rig:**  
**a – lay out, b- cross sections of thermocouples' location**

The rig consists of three similar blocks (Test sections 1-3). The main part of a section is a test copper tube 290 mm height, 10 mm outer diameter, 1 mm wall thickness. Each tube is mounted inside of water jacket of 180 mm diameter.

A water circulation contour consists of a pump, water piping, water tank, measurement and control systems including regulation and stop valves.

Circulating water was fed in parallel and supplied from the bottom of the jackets and removed from the upper part, so that an upward flow of water inside all jackets took place. The flow rate of water was controlled by a rotameter, adjusted and kept constant during every experimental run by precision needle valves, and precisely measured by the volumetric method. The water flow rate could be maintained individually for each test unit.

A given temperature at the entry to the jackets has been maintained by switching on / off of a cooling coil or electric heater in the water tank.

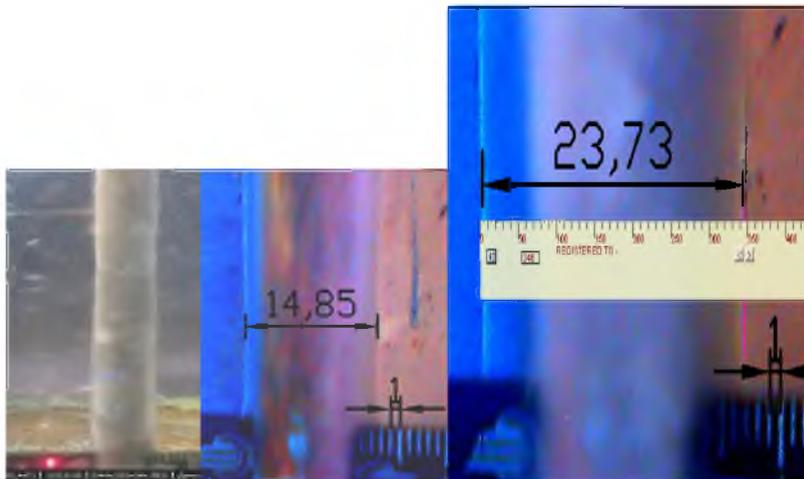
The test tubes were hooked up to the refrigeration (R12, R22) contour in parallel on the refrigerant. The refrigeration unit has been equipped with all necessary systems

allowing control, measurements and regulation of the evaporation pressure (evaporation temperature) inside each test tube and condensation pressure in the condenser.

A set of thermocouples installed in four equally spaced cross sections along the tube, see Fig.3 (b) allowed measurements of temperature of water at a distance 40 mm from the tube surface and tube outer surface temperature at the same cross section.

Time rate of ice layer thickness formed during the experiments was measured by means visual technique. Photographs of the test sections of pipes covered with ice were taken from the front of the transparent water jackets. Immediately before the experiments, an adjustment session had been carried out including testing of different lighting techniques and light sources, and calibration procedures which aimed at the determination of the best measurement arrangements.

Photographs were taken with a digital camera Canon 350 D 8.2 MP and simultaneously with a web camera HP HD-4110 (13 MP). Web cameras attached to each test section operated by the Active WebCam software which allowed taking pictures of the test pipes at any chosen frequency and storing individual video files for every section. Along with taking pictures with the web cameras the individual pictures were taken with the frequency 1 picture per 30 second with a Canon 350 D camera. Synchronization of the pictures taken by the Canon photo camera on the time scale of the web camera has been made by shooting a laser pointer beam on the test pipe simultaneously with taking picture with the Canon camera along with the continuous filming the process with the web camera. The picture made by the Canon camera with the red point collated with the individual shot of the web camera film with the same red point.



**Figure 4. Synchronization of individual cadres**

The measurements of iced pipe diameters have been performed by a set of on screen measurements software (ScreenRuler, PixRuler, Acad). All of them gave the results with the deviation of 0.2...0.4 mm.

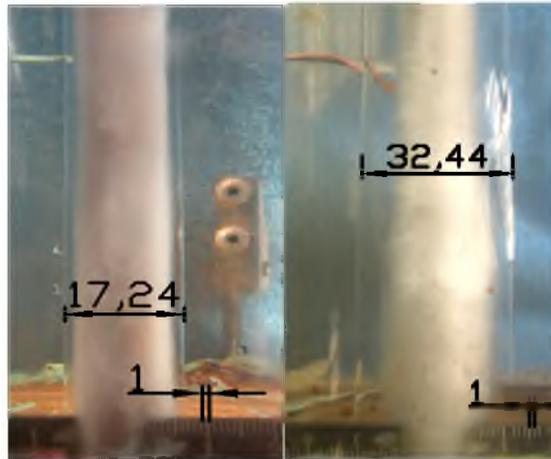


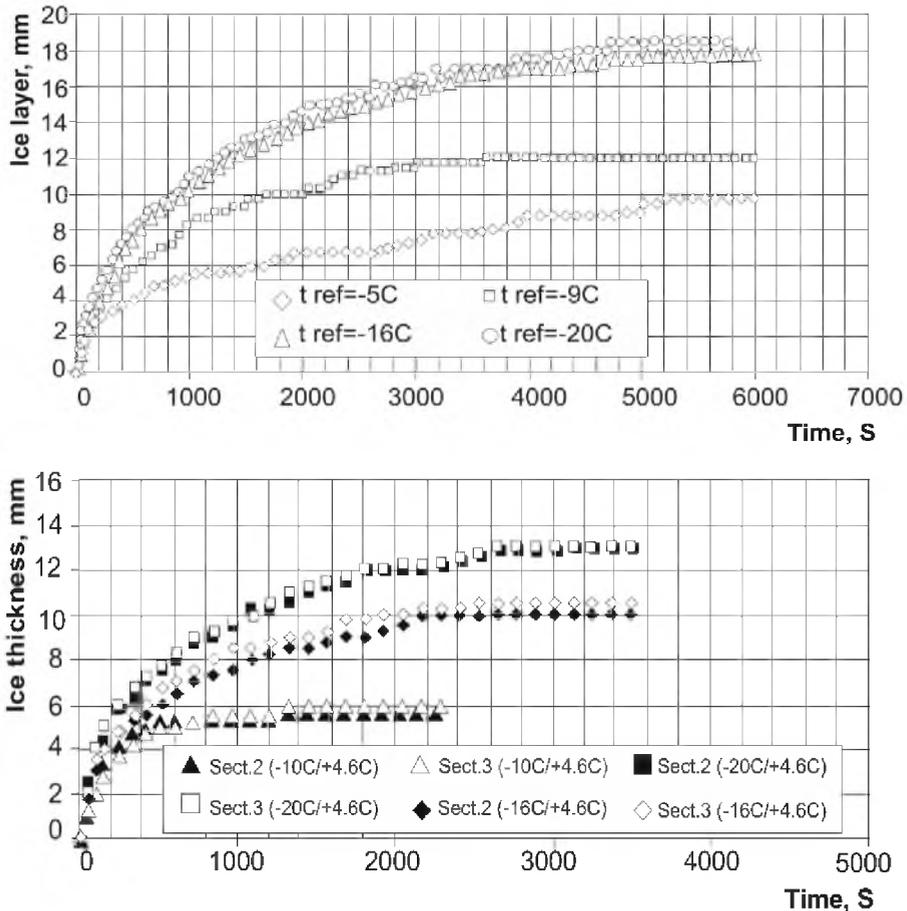
Fig. 5. Ice layer thickness measurements

## Results and discussions

Experiments were carried out at a fixed water flow rate at which corresponds to  $Re = 0 \div 16560$  and at three water inlet temperatures  $+1.5^{\circ}\text{C}$ ,  $+3^{\circ}\text{C}$ , and  $+5^{\circ}\text{C}$ . The temperature of evaporation inside test sections was kept constant within one run of experiments. The evaporation temperatures were kept at  $-5^{\circ}\text{C}$ ,  $-9^{\circ}\text{C}$ ,  $-15^{\circ}\text{C}$  and  $-20^{\circ}\text{C}$ . Visual observations show that ice formed at different temperature differences (water-evaporating refrigerant) looks different. At bigger temperature difference especially at higher water temperatures, ice formed is dim, non-transparent with rough porous surface, whereas the ice formed at moderate temperature difference and water temperature  $1.5 \dots 3^{\circ}\text{C}$  is dense, completely transparent, although the ice layer thickness reaches 30 mm and more. The surface of ice is glassy. The later is shown in Fig.5. The data arranged in series at a constant temperature (a-  $+1.5^{\circ}\text{C}$ , b-  $+4.6^{\circ}\text{C}$ ) of overflowing water are shown in Fig. 6.

It is quite apparent that each series of data tends to reach a certain asymptotic value, which could be termed as a terminal for a given temperature difference value of ice thickness. Since a growing ice layer acts as a gradually increasing thermal insulation, the terminal value of ice thickness reflects a heat balance at which a state of thermal equilibrium is achieved. In this state only heat transferred from the overflowing water on the ice-water interface can be transferred to the refrigerant. No additional ice may be formed after the state of equilibrium has been achieved.

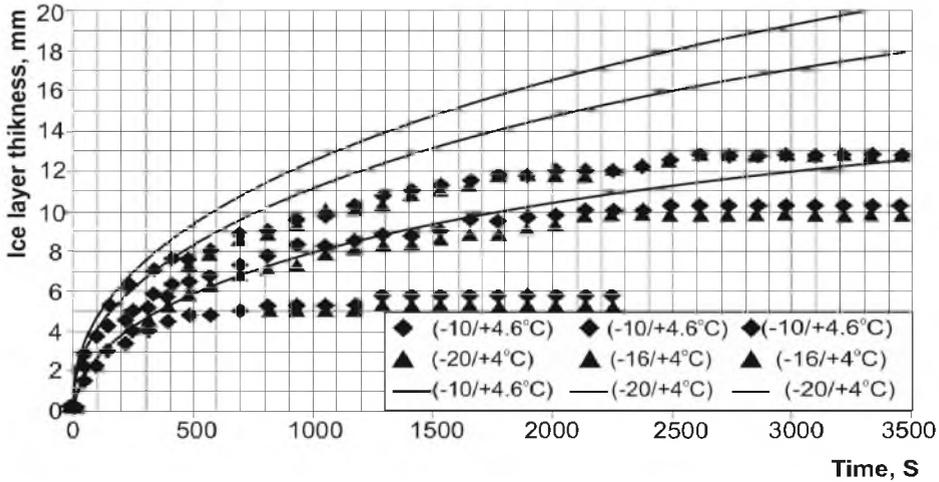
Similarly, the time intervals for reaching the terminal asymptotic values increase as the temperature difference increases. The asymptotic character of the experimental data allows choosing an optimal time interval of ice formation i.e. charging of a cold accumulator, since working after an asymptotic value of ice layer thickness has been reached leads to the direct loss of energy spent by the refrigeration compressors. Comparison of the data depicted in Fig.6 with the results of numerical solutions of the equation (5) with respective boundary conditions is given in Fig. 7.



**Figure 6. Time rate of ice formation:**  
 1st -  $t_{ref} = -5^{\circ}\text{C}, -9^{\circ}\text{C}, -16^{\circ}\text{C}, -20^{\circ}\text{C}, t_{water} = +1.5^{\circ}\text{C}$   
 2<sup>nd</sup> -  $t_{ref} = -10^{\circ}\text{C}, -16^{\circ}\text{C}, -20^{\circ}\text{C}, t_{water} = +4.6^{\circ}\text{C}$

The similar experiments have been conducted with the use R22 refrigerant within the temperature regimes that correspond to the experiments with R12 refrigerant. Given are the plots of experimental data with R22 which compared to the numerical solution of differential equation (5). The deviation from the given above plots is that the mentioned above solution have been carried out by 2 methods.

The first method of solution had been carried put at given boundary and initial conditions and was kept constant during a whole period of calculation. Therefore it could be termed as a continuous one. The second method had been carried out within a set of time intervals with the duration of 100...200 seconds. At the end of each time intervals the thickness of the build-up layer of ice had been determined, which in turn determined the reduction of interval heat transfer coefficient. This set of calculations allowed to determine a set of new boundary and initial conditions which were used for the calculation of differential equation of the following time interval. The solution thus obtained therefore can be termed as piecewise solution.



**Fig. 7. Comparison of experimental data and results of numerical solution of (5) Lines 1,2,3 link a numerical solution to the correspondent data set**

Therefore the proposed method of thermal resistance calculations in certain periods of time and substitution of the obtained data into differential equation allows to obtain more steep calculation curve which lays closer to the experimental data.

Given below the data which depict maximum ice-layer thickness at different feed-water temperature which obtained with 2 different frions. The experiment have shown that at similar regime parameters the series of experimental data have clear exponential character which characteristic to both Frions. This proves that the dynamic of generation on the external cylindrical vertical cooled surface at different  $\Delta t$  (temperature difference between that of water and boiling refrigerant) has exponential character which is clearly shown in figures given above.

As it follows from Fig.7 experimental data tend to deviate from the lines depicting numerical solutions and lie beneath the lines. This deviation can be explained by the inadequate estimation of the heat transfer coefficients  $h_r$  to evaporating Freon inside the pipe, since the local heat fluxes are far lower than those in experiments [7]. Unfortunately, there are no reliable recommendations on heat transfer calculations valid in conditions similar to those in our experiments. In order to further examine the developed model (5), a comparison has been done by application of the proposed model to the data [8]. The data [8] were obtained on the experimental rig which provided zero heat flux from the water to ice surface. Cooling of the pipe has been organized by flowing ethylene glycol cooled in a special heat exchanger. The same approach has been employed in [9]. These particulars allowed to exclude the effect of heat transfer coefficient  $h_w$  by putting it 0 in (5) and to determine  $h_r$  precisely by Dittus-Boelter correlation. Comparison of numerical solution with the data presented in [8] is given in Fig.10.

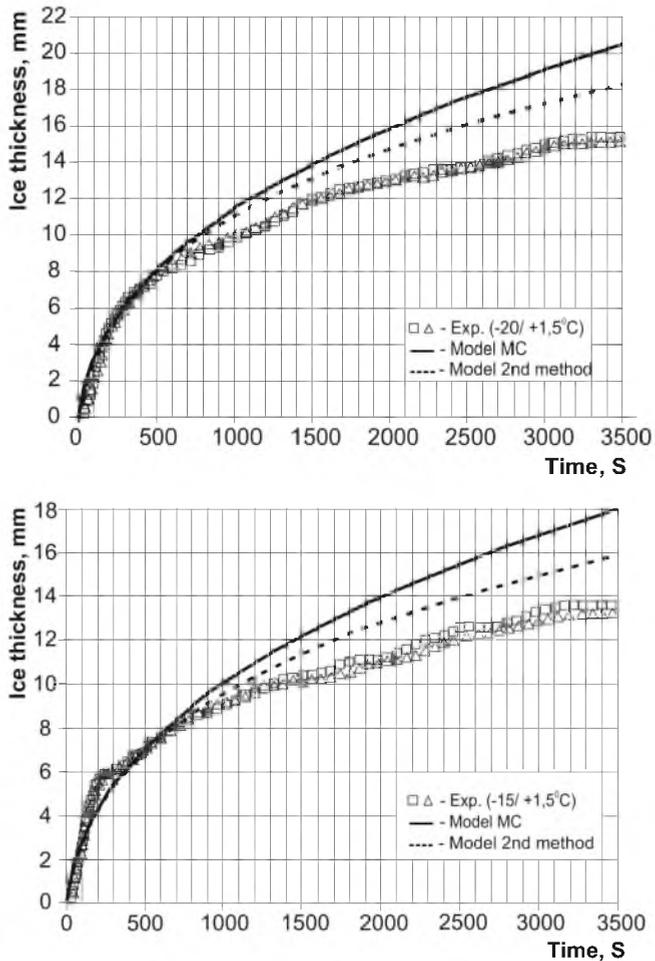


Fig. 8. Comparison of experimental data and numerical solution of equation (5)

As it is clearly seen from Fig.8, the results of calculations match closely experimental data [8].

This proves the adequacy of the proposed model but also witnesses that use of adequate values of heat transfer coefficients is critical for obtaining results matching those observed (5).

Since for the time being no reliable correlations for determination of heat transfer coefficients within the range of low heat fluxes and low refrigerant mass flow rates can be found, it seems advisable to formulate a simple semi empirical correlation in which a correction coefficient representing a ratio between the numerical solution and respective experimental result at the same ice layer thickness is determined. A correlation plot thus obtained is shown in Fig.11.

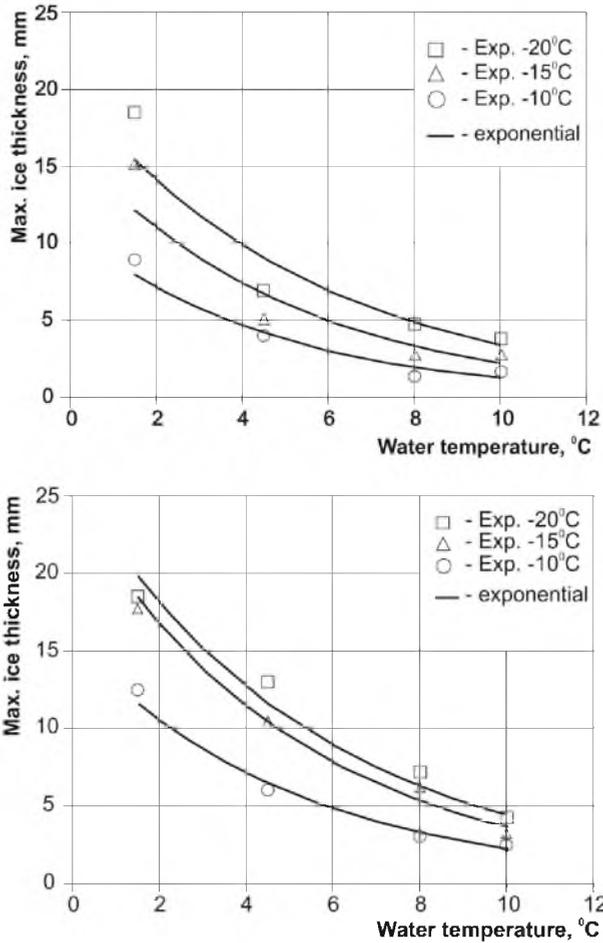


Fig. 9. Variation of maximum ice thickness with water temperature (for R-22 and R-12 respectively)

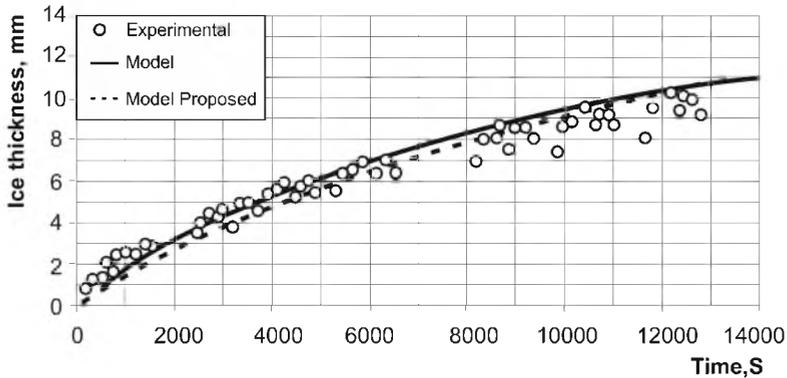


Fig. 10. Comparison of experimental data [8] with the results of calculation of (5). Cooling liquid inlet temperature-  $-4.3^{\circ}\text{C}$ .

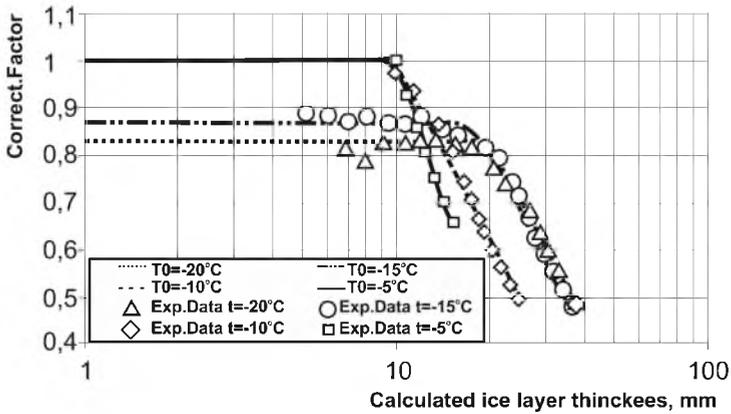


Fig. 11. Correction factor  $X_{\text{exper}} / X_{\text{num}}$  variation

## Conclusions

1. Close correspondence of the data available in literature with the results of numerical solution of (5) proves its adequacy in application to the processes for which heat transfer mechanism is well established and heat transfer coefficients may be calculated.

2. Deviation of experimental data obtained in the present work from the calculated curve is determined by the incorrect values of heat transfer coefficients of evaporating refrigerant (6). Since due to the decrease of heat transfer flux as a result of ice layer growth, nucleate pool boiling heat transfer coefficient of refrigerant decreases. This determines the fact that the thickness of ice layer reaches its asymptotic value within a comparatively short period of time.

3. When cooled by the flow brine at forced convection with a constant heat transfer coefficient, the period of reaching an asymptotic ice thickness is much longer.

4. Ice formation time rate on the pipes cooled by evaporating refrigerants may be determined by calculating a time curve by (5) for the respective conditions with heat transfer coefficients determined by (6) at first. By utilizing a correction factor determined by Fig.9, an actual time curve of ice buildup can be calculated.

## References

1. Fukusako S., Yamada M. (1993), Recent Advances in Research on Water-Freezing and Ice-Melting Problems, *Experimental Thermal and Fluid Science*, 6, pp. 90-05.
2. Kayansayan N., Ali Acar M. (2006), Ice formation around a finned-tube heat exchanger for cold thermal energy storage, *International Journal of Thermal Sciences*, 45, (4), pp. 405-418.
3. Pylipenko O.Yu., Zasiadko Ya.I. (2008), Substantiation of the feasibility of latent heat cold accumulators application, *Refrigeration techniques and technology*, 5(115), pp. 11-15 [In Russian]
4. Pylipenko O.Yu., Zasiadko Ya.I. (2010), Options of energy demand optimization in the artificial cold generation, *Food Manufacturing equipment and technologies. Collection scientific proceedings*, 24, pp. 54-62 [In Russian]

5. Vargas J.V.C., Bejan A. (1995), Fundamentals of ice making by convection cooling followed by contact melting, *International Journal of Heat and Mass Transfer*, 38(15), pp. 2833-2841.
6. Chumak, I.G. (1995), *Refrigeration Plants*, Lybid Edition, Kyiv
7. Danilova G.N., Bogdanov S.N., Ivanov, O.P. (1973), *Heat exchangers of Refrigeration Plants*, Leningrad [In Russian]
8. Yilmaz A., Fertelli A., Buyukalasa O. (2009), Ice formation around a horizontal tube in a rectangular vessel, *Journal of Thermal Science and technology*, 29(2), pp. 75-87.
9. Mousa M. Mohamed (2003), Solidification of Phase Change Material on Vertical Cylindrical Surface and Stored Thermal Energy in Holdup Air Bubbles Columns, *Engineering Research Journal*, 26(4), pp. 1-24.
10. Yoon J.I., Moon C.G., Kim E., Son Y.S., Kim J.D., Kato T. (2001), Experimental study on Freezing of Water with super cooling region in a Horizontal Cylinder, *Applied Thermal Engineering*, 21(6), pp. 657-668.
11. *Heat exchangers Design Handbook* (1983), Hemisphere Publishing Corp, New-York.