# The Existence of Liapunov's Function for the Invariant Manifolds Differential Equations

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We concider the system of differential equations

$$\frac{dx}{dt} = X(t,x) \tag{1}$$

for $t \geq 0$, $x \in D \subset \mathbf{R}^n$, the function $X(t,x)$ is conti-nious on $t$ and $x$ and satisfies the Lipshitz condition on $x$. Let $M \subset \mathbf{R}^{n+1}$, and $M_{t_0}$ is the intersection of $M$ with the hyperplane $t = t_0$, $t_0 \geq 0$. The following theorem holds true.

**Definition.** The set $M$ is called the positively invariant set of the system (1), if the solution $x(t)$ of the system (1), such that $x(t_0) \in M_{t_0}$ has the property: $x(t) \in M_t$, $\forall t \geq 0$.

**Definition.** The positively invariant set $M$ of the system (1) is called stable for $t \geq t_0$, if $\forall \varepsilon > 0\ \exists \delta = \delta(\varepsilon, t_0) > 0$ such that $\rho(x(t_0), M_{t_0}) < \delta$, then $\rho(x(t), M_t) < \varepsilon$ for $t \geq t_0$.

**Theorem.** Let the system (1) for $t \geq 0$ have a smooth, positively invariant stable manifold $M$ such that $Pr_{\mathbf{R}^n} M$ is compact in $D$ ($Pr_{\mathbf{R}^n} M$ is the projection of the set $M$ on $\mathbf{R}^n$). Then, in the domain $t \geq 0$, $x \in D$ there exists the Liapunov's function $V(t,x)$, which is differentiable in any direction and has the following properties:

1. $V(t,x)$ is positively defined uniformely on $tgeq0$, that is

$$\inf_{t\geq 0;\, x:\rho(x,M_t)>\varepsilon} V(t,x) = V_\varepsilon > 0, \qquad \forall, \varepsilon > 0;$$

2. the derivative of the function $V(t,x)$ with respect to the system (1) is non-positively defined:

$$\frac{\partial V}{\partial t} + \left(\frac{\partial V}{\partial x}, X(t,x)\right) \leq 0, \qquad t \geq 0, \qquad x \in D;$$

3. the set of zeros of the function $V(t,x)$ is $M$, that is

$$M = \{(t,x) : V(t,x) = 0, t \geq 0, x \in D\}.$$