Non-linear resonance vibrations of cracked beams in condition of driving force parameters variation

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ABSTRACT

A closing crack causes the dynamic behavior of a vibrating system to be significantly non-linear and, consequently, the appearance of non-linear resonances. The main idea of the proposed procedure of crack location and size estimation is based on the determination of the vibration response non-linearity around the superharmonic resonance of order 2/1 and subharmonic resonance of order 1/2 at different parameters of driving force. First parameter under investigation is the point of force application. Second parameter is the level of driving force asymmetry. It is shown both numerically and experimentally that the abrupt change of the non-linearity of vibration response when driving force is applied close to the crack clearly indicates the location of crack. Addition of the static component to the harmonic driving force varies the level of the vibration response non-linearity at any non-linear resonance in case of crack presence. Based on the variation of the driving force parameters the procedure of damage detection is proposed.

INTRODUCTION

Fatigue cracks are the most widespread damage of dynamically loaded structural elements. Usually it is supposed that such cracks periodically close and open in the process of cyclic deformation of a body, leading to the instantaneous change of its stiffness. The change of stiffness as a rule is modeled by the piece-wise linear characteristic of the restoring force [1] or by the specific modification of the driving force [2].

A closing crack causes the dynamic behavior of vibrating system to be significantly non-linear. Numerical investigations of forced vibrations of beams with a closing crack [3, 4] have demonstrated that the main distinctive features of such a vibration system are the manifestation of effects associated with non-linearity, namely the presence of super- and subharmonic resonances and significant non-linearity of the vibration response at super- and subharmonic resonances (so called non-linear resonances) of different orders. The sensitivity of non-linear effects to a crack presence exceeds many times the sensitivity of natural frequencies and mode shapes [3-5]. This opens up the perspective of using super- and subharmonic resonance regimes for the detection of fatigue cracks.

Manifestation of the non-linear effects depends not only on the crack parameters (that is crack size and location) but also on the level of damping in a vibrating system [5, 6] and on the power of excitation [6, 7]. The data of direct experimental investigations [8-11] attest that the fatigue crack growth is accompanied by a considerable increase of damping characteristic of specimens. Consequently, the determination of the relationships between the crack parameters and the non-linear effects must be realized while taking into account the change of damping in a vibrating system rather than assuming constant damping which has been the case in studies [2-4].

In previous investigations of forced vibrations of damaged bodies the driving force parameters and the level of damping were supposed to be unchangeable. However, as our investigations showed, the non-linear effects are strongly dependent on the driving force application point and driving force asymmetry. This opens up the perspective for the development of new methods of damage detection based on the determination of the non-linearity of vibration response at non-linear resonances in condition of variation of driving force parameters. Such an approach makes it possible to cut off the influence of other factors (such as non-linear damping, geometrical non-linearity etc.) which can influence the non-linear effects.

So the aim of the work was to investigate the effect of the driving force application point and driving force asymmetry on the non-linearity of vibration response at super- and subharmonic resonances as applied to the detection of cracks in beams taking into account the cased by a crack change of damping.

MODEL OF A CRACKED BEAM

The mathematical model of a beam with an edge transverse closing crack is based on the finite element model (FEM) proposed in ref. [3]. While as the crack is closed and its interfaces are completely in contact with each other, the dynamic response can be determined directly as that one of the intact beam; while as the crack is opened the stiffness matrix of the cracked element should be introduced in replacement at the appropriate rows and columns of the general stiffness matrix. It was suggested that the system has the piecewise-linear characteristic of the restoring force. The vibration of a cracked beam was described by two linear differential equations in normalized co-ordinates:

$$\begin{cases} [I]\{\ddot{q}\} + [\Lambda]\{\dot{q}\} + [\omega^2]\{q\} = \{R\}F(t); \\ [I]\{\ddot{q}\} + [\Lambda_d]\{\dot{q}\} + [\omega_d^2]\{q\} = \{R_d\}F(t), \end{cases} \tag{1}$$

where the matrices of a beam with a closed crack are shown without subscripts and the matrices of a beam with an open crack are shown with subscript d (damaged beam); [I] is the mass matrix (unitary); $[\omega^2]$ is the stiffness matrix; $[\Lambda]$ is the damping matrix; [R] is the external load vector; F(t) is the external load function; {q} is the normal coordinates. The first equation describes the vibration of a beam with a closed crack and the second one - of a beam with an open crack. These equations were solved with the Runge-Kutta method proceeding step-by-step in time. In the process of the solution of Eqs. (1) the check of the crack state is realized by the comparison of the angles of rotation in the nodes from the right and from the left of the cracked element.

If during the i-th time step the state of a crack changed, then the initial conditions for the next (i+1) time step should be calculated by the following formulas:

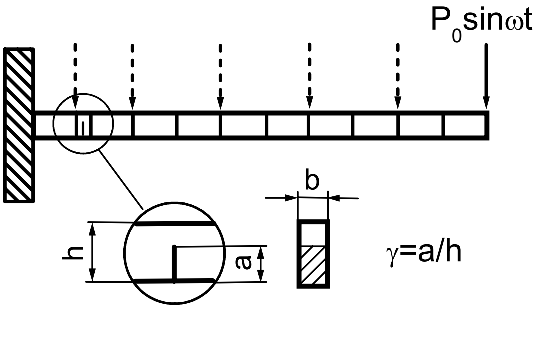

Figure 1. The finite element mesh for a beam

$$\{q_{i+1}\} = [\Phi_d]^{-1}[\Phi]\{q_i\}; \tag{2}$$

$$\{q_{i+1}\} = [\Phi]^{-1}[\Phi_d]\{q_i\}, \tag{3}$$

where $[\Phi]$ is the matrix of mode shapes of the beam with a closed crack; $[\Phi_d]$ is the matrix of mode shapes of the beam with an open crack. Formula (2) is used for the moment of crack opening and formula (3) – of crack closure. The mesh with a local concentration in the area of crack location is used for the increase of accuracy of the determination of crack location (Fig. 1). To calculate the equivalent stiffness of this element the strain energy balance method is used [5].

EXPERIMENTAL PROCEDURE

The tests were performed using the original experimental set-up KD-1M [5]. This set-up has been devised for the determination of damping characteristic, for the high-cycle fatigue testing of specimens, and for the modal analysis of specimens. The specimens were fixed in the rigid frame (Fig. 2). The electromagnetic system which

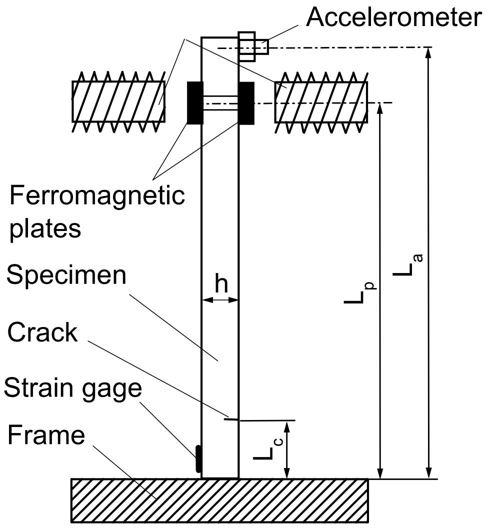

Figure 2. Schematic sketch of the specimen

consisted of two electromagnets, computer-based waveform generator and power amplifier was used for the excitation of the first bending mode of vibration. The special ferromagnetic plates were attached to the specimens to create the interaction between the specimen and electromagnets. Fatigue cracks were grown from sharp concentrators in fatigue tests. The measurements of the cracks depth were executed by an optical microscope, the absolute error being ±0.1 mm. The damping characteristic of cracked specimens δ (logarithmic decrement of vibration) was determined by the free damped oscillation method.

The titanium alloy VT-3 and carbine Steel 3 specimens were tested. The dimensions of the specimens and mechanical properties of the materials are shown in Table 1, where L is the length of the specimens; h and b are the height and width of the cross-section, respectively; E is Young's modulus; ρ is the density; σ_{-1} is the fatigue limit. The parameters of crack size (a) and location (L_c), as well as the location of accelerometer (L_{ac}) and driving force application points (L_P) are given in Table 2 (δ is the logarithmic decrement of vibration).

The preliminary tests showed that even small fatigue cracks cause the significant increase of damping for both specimens. So it is quite reasonable that this change of damping should be taken into account. Therefore the data these experiments were used to assign the level of damping in the FEM of cracked beam in calculations of the non-linear effects.

Table 1. Dimensions of specimens and mechanical properties of materials

Material of the specimens	L, [mm]	h, [mm]	b, [mm]	E, [GPa]	ρ, [kg/m^3]	σ_{-1}, [MPa]
VT-3	230	20	4	110	4480	480
Steel 3	230	19.5	4	200	7800	190

Table 2. Parameters of tests at non-linear resonances

Specimens	Non-linear resonance	a, [mm]	L_c, [mm]	L_{ac}, [mm]	L_{P1}, [mm]	L_{P2}, [mm]	δ
VT-3	superharmonic	2.1	11	223	180	96	0.009
Steel 3	subharmonic	8.0	14	223	182	99	0.020

It is common practice to investigate analytically or numerically the forced vibrations of different systems based on the assumption that the driving force is harmonic. However it is extremely difficult in practice to generate rigorously harmonic excitation. So if the frequency of the driving force is several times lower than the frequency of principal resonance, the vibration response may appear similar to the superharmonic regime of vibration (so called pseudo-superharmonic resonance). Such vibration takes place while as the frequency of one of the high harmonics in the spectrum of driving force coincides with the frequency of principal resonance. To solve this problem the PC-based technology was used. The initial signal was generated with the use of special algorithm by the computer-based waveform generator of the excitation system in such a way that its subsequent distortions by the power amplifier and by the electromagnets compensated the initial one. It is necessary to note, that on excitation of the subharmonic resonance there is no need of such a thorough monitoring of the driving force non-linearity.

EXPERIMENTAL VALIDATION OF THE MODEL

The results of two specimens test were used to validate the presented above FEM of a beam. The estimation of validity of the model was executed by the comparison of numerical and experimental results on the change of natural frequency and on the non-linear effects for the cantilever prismatic beams. First of all the proposed FEM can quite accurately predict the change of the first bending natural frequency of beams with an edge fatigue crack of different depth: the discrepancy between the results of calculations and experiments did not exceed 4%.

The level of the vibration response non-linearity at both non-linear resonances is very high. That is why these resonances were investigated not in terms of amplitude of vibration (this amplitude is very small from practical point of view) – relative frequency of driving force, but in terms of vibration response non-linearity – relative frequency of driving force.

Experimental and predicted dependencies of the acceleration response non-linearity for the alloy VT-3 and Steel 3 specimens from the relative frequency of the driving force at super- and subharmonic resonances are shown in Fig. 3. The results of experiments are indicated by dots. The resonance curves were determined for two different points of load application. The first point was close to the end of the beam, and the second point was approximately in the middle (Table 2). The experiments were carried out in the range of very small stress amplitudes (σ_a=5…15 MPa) to avoid a crack growth in measurements of the non-linear effects.

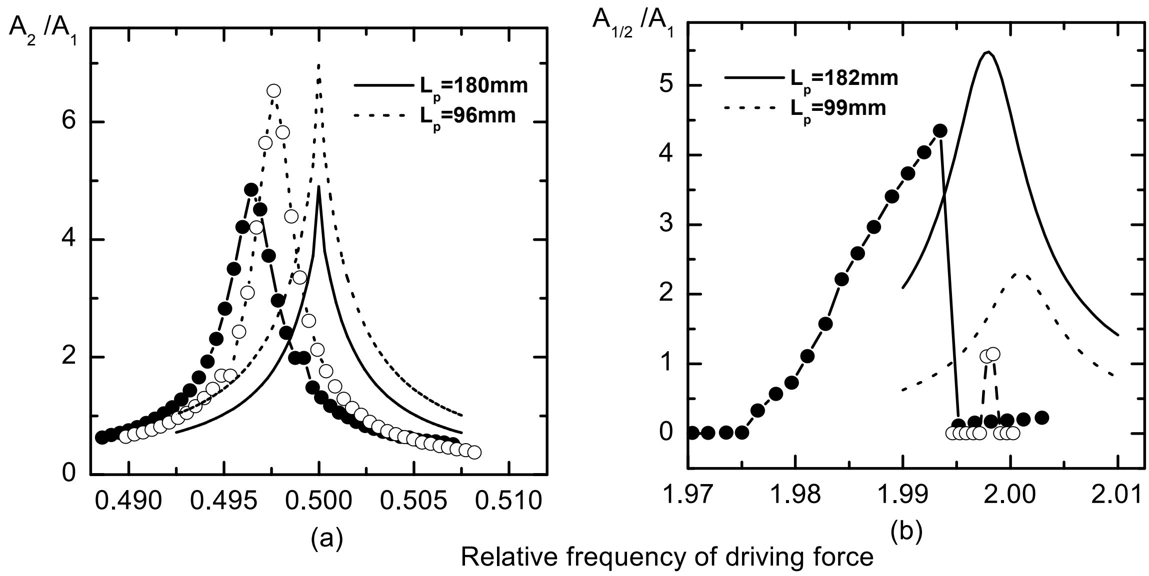

Figure 3. Relative frequency of driving force dependencies of acceleration response non-linearity at super-harmonic of order 2/1 (a) and subharmonic of order 1/2 (b) resonances for the alloy VT-3 (a) and Steel 3 (b) specimens (results of experiments are indicated by dots)

First of all it is important to note that the non-linearity of vibration response at super- and subharmonic resonances is very sensitive to a crack presence. The sensitivity of this characteristic is one-three order of magnitude higher than the sensitivity of natural frequencies or mode shapes. Secondly, this non-linearity is dependent on the point of force application. The influence of the force application point is qualitatively different at superharmonic and subharmonic resonances: in the first case the non-linearity of acceleration response is lower at upper point of force application than at middle one, in the second case – high. As can be seen, qualitatively the predictions of the model are in a good agreement with the results of experiments.

At the same time, the tests revealed two distinctive features of subharmonic resonance which were not predicted by the FEM. First one: its appearance is strongly dependent on the power of excitation. Under the insufficient power of excitation the subharmonic resonance does not appear even in presence of big crack. To excite this regime of vibration it is necessary to apply the power exceeding considerably the power necessary for excitation of superharmonic resonance. The second feature is that the subharmonic regime of vibration is extremely unsteady: it can arise and disappear unexpectedly.

In spite of the fact that the FEM can not reveal these features of subharmonic resonance, and therefore requires certain improvement, it can predict sufficiently good the effect of the driving force application point on the non-linearity of vibration response and the level of non-linearity of vibration response at both non-linear resonances. The FEM also correctly predicts the different sensitivity of sub- and superharmonic resonances to the crack presence.

EFFECT OF DRIVING FORCE APPLICATION POINT: RESULTS OF CALCULATIONS

So the proposed FEM of a beam was used to investigate the non-linear resonances of beams with different boundary conditions that is: cantilever, simply supported and constrained beams. The dimensions of the cantilever beam were L=0.2m; h=0.02m; b=0.004m. The dimensions of the simply supported and constrained beams were L=6m; h=0.5m; b=0.5m. The Young's modulus, the density and the decrement of vibration of beams material were E=206Gpa, ρ=7850kg/m^3 and δ=0.01, respectively. In all cases the crack size and location were a/h=0.2 and L_c/L=0.5.

The investigations were executed in the following way: the force was applied to different nodes of the FEM step by step, as it was shown in Fig. 1. At every step the spectral analysis of the acceleration response at a non-linear resonance of a beam was calculated. The vibration response was related to the end of a cantilever beam, and to the middle of simply supported and constrained beams.

As can be seen from Fig. 4, the non-linearity of vibration response of beams at both non-linear resonances significantly depends on the driving force application point. The sharp local break of these curves distinctly reveals the crack location (L_c/L=0.5). In addition, the level of non-linearity makes it possible to judge about the crack size, at least on condition that the damping characteristic of the cracked beam is known.

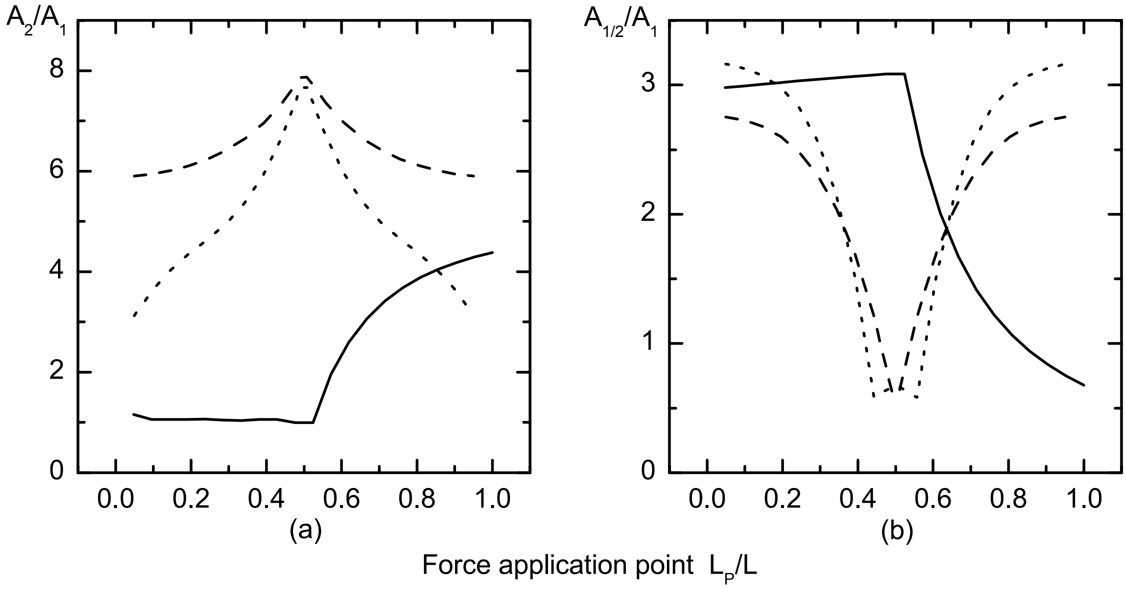

Figure 4. The amplitudes of dominant harmonics vs. the co-ordinate of the force application point: superharmonic of order 2/1 (a) and subharmonic of order 1/2 (b) resonances for the cantilever (solid line), simply supported (dash line) and constrained (dot line) beams

For the simply supported beam the crack location discloses itself by the peak of non-linearity. At superharmonic resonance this is a maximal and at subharmonic resonance this is a minimal level of the non-linearity. Practically the same results were obtained for the constrained beam.

It is interesting, that if a crack will locate in the point L_c/L=0.25 the subharmonic resonance does not appear at all. The reason of this phenomenon is the fact that the node of bending stress at the first mode of vibration for the constrained beam is very close to a crack location (0.223) and the non-linearity of the system was insufficient to launch the subharmonic resonance. At the frequency of subharmonic resonance the non-linearity of vibration response shows up only if the crack reaches a certain definite size. Since that value the damage characteristic shows high sensitivity to the crack presence.

In general, one should remember that a closing crack located in any node of stress (it does not matter at bending or axial vibrations) can not be detected neither by the non-linear effects nor by the change of damping or natural frequency. To increase the reliability of vibration based methods of damage detection it is necessary to explore several mode shapes.

EFFECT OF DRIVING FORCE ASYMMETRY: RESULTS OF CALCULATIONS

The FEM of the simply supported beam was used also for the calculation of the vibration response non-linearity at super- and subharmonic resonances in conditions of variation of the crack state. For that the static component P_{st} was added to the harmonic driving force $P=P_0\sin\omega t$. Addition of the static component to the harmonic driving force makes crack more or fully open or closed changing in such a way its state. As a result the level of the non-linearity of vibration response at any non-linear resonance should vary from maximal value (in absence of static component) to practically zero value (when the static component of driving force is so large that crack becomes permanently open or closed at vibration).

The parameters of the beam were the same as in the previous section. The static component of the driving force was applied in the middle of the beam and the harmonic excitation was applied in the point $L_p/L=0.5$ or $L_p/L=0.25$. The acceleration response was related to the middle of the beam.

As can be seen from Fig. 5, the non-linearity of acceleration response at super- and subharmonic resonances essentially drops with the increase of the absolute value of static component. In the comparatively wide range of the ratio P_{st}/P_0 the non-linearity decreases slowly, then abruptly drops to zero value. Qualitatively the effect of the crack state on the non-linearity at both non-linear resonances is the same, but the level of non-linearity at subharmonic resonance in this particular case is much lower. It is necessary to bear in mind that at other crack and driving force parameters the subharmonic resonance may be more sensitive than superharmonic one.

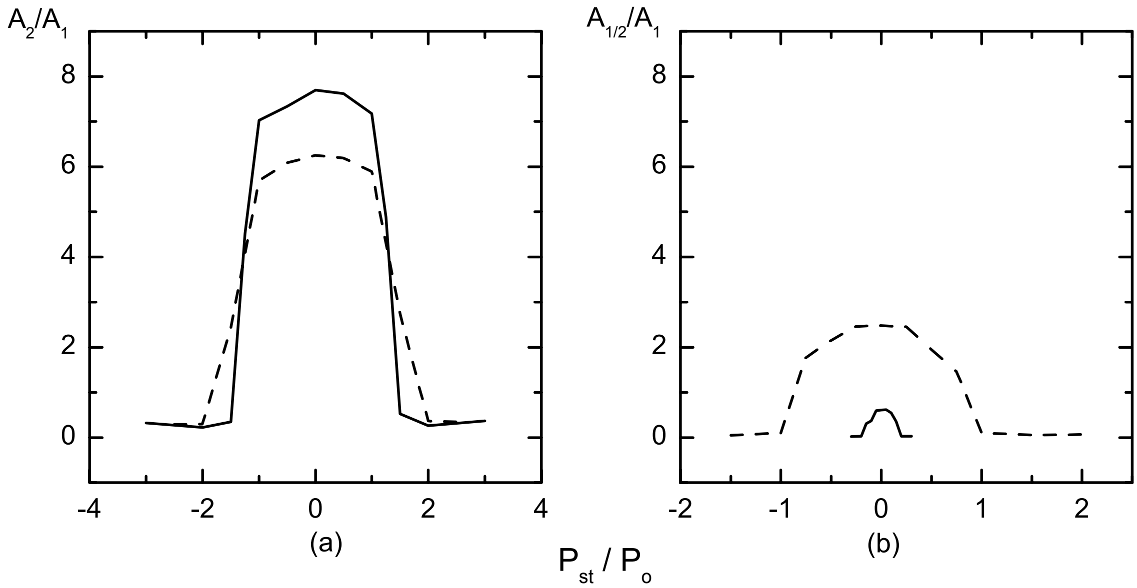

Figure 5. The effect of crack state on the acceleration response non-linearity at superharmonic of order 2/1 (a) and subharmonic of order 1/2 (b) resonance for the cracked simply supported beam at driving force application point $L_p/L=0.5$ (solid line) and $L_p/L=0.25$ (dash line)

In such a way the presence of crack can be detected without any preliminary information on the vibration response of a structure in an intact state. If the addition of the static component results in the change of the vibration response non-linearity, one can definitely suppose the presence of a crack. If there is no change of the non-linearity, no crack can be expected.

CONCLUSIONS

The non-linearity of vibration response around the sub- and superharmonic resonances of beams is very sensitive to the presence of a closing crack. This non-linearity, as was shown both experimentally and numerically, is strongly dependent on the crack size and location, damping level, driving force application point and driving force asymmetry.

The excitation of vibration of a beam by the application of driving force to different points along the beam resulted in the essential change of the vibration response non-linearity at super- and subharmonic resonances. The abrupt change of this non-linearity clearly indicates the crack location. At the same time the level of vibration response non-linearity is directly proportional to the crack size. Consequently, the measurements of the vibration response non-linearity around the super- and/or subharmonic resonances applying the driving force in several points along the beam enable to determine the crack parameters by the proposed FEM of a beam. The accuracy of the crack size determination depends on the accuracy of the damping characteristic estimation.

One more damage detection procedure is proposed. Its essence consists in the artificial variation of the crack state by the addition of the static component to the harmonic driving force. The change of vibration response non-linearity at super- or subharmonic resonances in the course of such a variation can serve as an indicator of crack presence. The advantage of the procedure is that no preliminary information on the vibration response of a structure in an intact state is required.

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