APPLICATION OF ORDINARY DIFFERENTIAL EQUATIONS IN SOLVING DIFFUSIVE PROBLEM

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Abstract: *It is investigated functional dependence, which expresses changing's of concentration of expanded fluid in thickness of diffusive stratum. A solution of the given problem has been received as a solution of an ordinary second-order differential equation i.e. as a solution of Cauchy problem.*

Keywords*: ordinary second-order differential equation, diffusion, chemical reaction, general solution, Cauchy problem.*

Анотація: *Досліджено функціональну залежність, яка виражає зміну концентрації розчиненого газу по товщині дифузійного шару. Отримано розв'язок даної задачі у вигляді розв'язку звичайного диференціального рівняння другого порядку, а саме розв'язку задачі Коші.*

Ключові слова: *диференціальне рівняння другого порядку, дифузія, хімічна реакція, загальний розв'язок, задача Коші.*

Mathematical models of many engineering and technological processes can be equated to a single differential equation or to their system. For instance, studying kinetics of ion exchange in processes of flowing extraction, filtering of liquid, analyzing settling of solid particles the significant results were brought by wide application of differential equations. Therefore qualitative theory of differential equations is an effective mathematical instrument for describing of phenomenon in many fields. The usage of differential equations defines their practical value. Owing to their usage it is possible to set up a connection between a basic physical or chemical law and often a whole group of variables, which have major significance studying the food processes**.**

This article is dedicated to the construction of the mathematical model of the diffusion problem which is accompanied by a chemical reaction. The diffusion's speed in the liquid is proportional to the concentration gradient.

We consider a diffusion layer of liquid which bounds with an interphase bound fluid - liquid. It is necessary to find the functional dependency which expresses a change of concentration of soluble fluid in thickness of the diffusion stratum.

Conditions of the process are the same in any plane that is perpendicular to the direction of diffusion. Let us separate in the bounded layer an element with its thickness dx. This element is bounded by planes that are parallel to the plane of distinction of phases and they have been constructed at the distance x and $x + dx$ from this plane. We should make a material balance for this element. The area of this element is assumed to be equal to the unity.

The diffusion rate at points of the plane that is x distant from the plane of distinction of phases will be equal to $\left(-D\frac{dc}{dx}\right)$, where D is the diffusion coefficient, c is concentration of fluid in liquid at the depth of x. Since concentration decreases in the direction of the diffusion flow then the diffusion coefficient should be taken with the negative sign.

Quantity of passing fluid through this plane during the time $d\tau$ is $\left(-D\frac{dc}{dx}d\tau\right)$.

Likewise, quantity of passing fluid through an opposite bound of the elementary layer that is $x + dx$ distant from the plane of distinction of phases will equal:

$$-D\left[\frac{dc}{dx} + d\left(\frac{dc}{dx}\right)\right]d\tau,$$

since the concentration gradient in this plane

$$\frac{dc(x+dx)}{dx} = \frac{dc}{dx} + d\left(\frac{dc}{dx}\right).$$

Fluid interacts with liquid in time of the diffusion through an elementary volume. The rate of this interaction is proportional to its quantity which this layer contains. Note the volume of the considered element equals dx, it implies the quantity of the passing substance through the elementary volume will be obtained by multiplying of this volume by concentration c. Nevertheless the rate of the chemical reaction is proportional to concentration, therefore it is equal to kc, that is: $-\frac{dc}{d\tau} = kc$, where k is a constant of the rate of the reaction.

Thus, quantity of passing fluid that enters into the chemical reaction in the elementary volume dx for the time $d\tau$ becomes equal to the product $kc\ dx\ d\tau$.

Therefore an equation of the material balance will have the following form:

$$-D\frac{dc}{dx}d\tau + D\left[\left(\frac{dc}{dx}\right) + d\left(\frac{dc}{dx}\right)\right]d\tau - kc\ dx\ d\tau = 0.$$

After simplifying we will obtain $\frac{d^2c}{dx^2} = \frac{k}{D}c$.

For solving of this equation we substitute $\frac{k}{D}$ for a^2 then it will be got the following form

$$c''(x) - a^2 c(x) = 0. \qquad (1)$$

Equation (1) is an ordinary second-order differential equation with constant coefficients. Whereas the look of roots of the characteristic equation, we will receive a general solution of equation (1):

$$c(x) = k_1 e^{ax} + k_2 e^{-ax}. \qquad (2)$$

Assume the concentration of fluid in the bounded layer and in the layer that is l distant from the bounded one be known. In other words, $c(0) = \omega,\ c(l) = \psi$. Solving the system of equations $\begin{cases} \omega = k_1 + k_2, \\ \psi = k_1 e^{al} + k_2 e^{-al} \end{cases}$ concerning constants k_1 and k_2 we will get

$$k_1 = \frac{\psi - \omega e^{-al}}{2\, shal};\ k_2 = \frac{\omega e^{al} - \psi}{2\, shal}.$$

Therefore, the solution of equation (2) will have the following form

$$c(x) = \frac{\psi\ shax + \omega\ sha(l-x)}{shal} = \frac{\psi\ sh\sqrt{\frac{k}{D}}x + \omega\ sh\sqrt{\frac{k}{D}}(l-x)}{sh\sqrt{\frac{k}{D}}\,l}.$$

Thus, a Cauchy problem has been solved.

Conclusion: The functional dependence has been found, which shows a change of concentration of soluble gas in thickness of the diffusion layer in the form of a second-order differential equation with constant coefficients. In case, concentration of gas in the boundary layer and in the layer at l distance from the boundary one is known, then a solution of Cauchy problem has been received.