10th International Conference
"Research and Development in Mechanical Industry"
RaDMI 2010
16 - 19. September 2010, Donji Milanovac, Serbia

MODELLING OF FOOD PRODUCT CUTTING

Viktor Guts[1], Oleksiy Gubenia[1], Stefan Stefanov[2], Wilhelm Hadjiiski[2]
[1]National university of food technologies, Kyiv, UKRAINE
[2]University of food technologies, Plovdiv, BULGARIA
gubena@meta.ua, goots@ukr.net, stvstefanov@yahoo.com, hawi@abv.bg

Summary: *There are lead theoretical experimental researches on the basis of which is developed the simple in application technique for research of movement of an edge in a layer of a product. The mathematical model which adequately describes process is also developed. The technique is applied at research of bread cutting by results of which optimum cutting conditions are certain.*

Keywords: *bread, bread cutting, optimum cutting*

1. INTRODUCTION

Cutting is a commonly used process for the preparation of raw materials for food production. Recently, many of the ready-to-use food products are subjected to cutting. This is related to their ease use and the willingness of the consumers. In some cases the quality of the finished food product depends from the quality of the cutting process.
Cutting technology of the food products is complicated. This is due to the imperfection of the process cutting, the characteristics of physical and mechanical properties of raw materials and foodstuffs, the nature of the stresses and strains that occur during the cutting process in the product. An example is the bread. Before cutting it has to stand for 2 to 4 hours. In that time it lost some of its properties and especially the freshness. On standing it requires a lot of space, as well as additional equipment, which leads to additional energy losses. The fresh bread, in the process of cutting, deforms and frictions to the knife and sticks to the knife surface. The problems that arise in the process of cutting require the modern solutions, enabling to improve the quality of the finished product and reduce energy losses and will not need to wait. So far there are not enough theoretical developments and accumulated experimental data that adequately characterize the process of cutting.
It is known that the process of cutting the elastic viscous products influences the speed of cutting. At high speeds of the blade is possible the reduction of the cutting forces. Moreover, reducing distortions of the product below the edge of the blade of the knife, it enhances the quality and productivity of the process.
Surveys and studies done for the study of problems with the cutting of food products show that now doesn't exist the methodology for cutting force measurement.
The purpose of this development is the creation of cutting process theory for elastic-viscous food products, methods for conducting experimental work to study the process and the development of specific experimental equipment for this purpose.

2. ANALYTICAL MODEL OF CUTTING PROCESS

We will discuss the mechanism of operation of the process of cutting. For this purpose it would be created a differential equation describing the movement of the knife blade in a layer of product.
On the blade, which penetrates the product, acts the following resistance forces (Fig. 1): P_i - inertia force, F_r - cutting force, G - force of friction between the side of a knife and product F_{ad} – adhesion force (depending on structural and mechanical properties of the product). In a small adhesion strength of the product prevails friction

force; and opposite in a sticky products – predominant is adhesion force. Frictional force is determined by the formula [1]:

$$G = C_{mp} + k_1 V = C_{mp} + k_1 \frac{dy}{dt}, \tag{1}$$

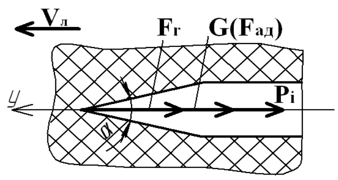

Figure 1: Scheme of the forces*, which acts on the knife*

where C_{mp} – coefficient which depends on the relative load on the product over side surface of the knife; V – velocity slip between the product and the side surface of the knife; k_1 - factor which takes into account the influence of sliding speed of the frictional force; y – displacement of the knife in the product; t – time of the blade movement in the product.

For the products that stick over the knife is necessary to take into account the power of the adhesion. It depends on the shape and surface condition of the blade, cutting speed, the area of the contact angle of cutting, structural and mechanical properties of the product and some other factors. The strength of adhesion is determined by a new methodology [2], or using literature data.

We could write the adhesion power as a function of the adhesion strength Pad at normal separation mechanism of the blades from the surface of the product:

$$F_{ad} = P_{ad} S$$

(2)

where P_{ad} – adhesion strength, Pa; S – contact surface, m^2.

Having in mind that separation is under an angle:

$$F_{ad} = P_{ad} S \cos(\alpha + \alpha_0)$$

(3)

where α – sharpening angle of the knife; α_0 – characteristic angle of separation could be both positive and negative. It shoud be noted that:

$0 < \cos(\alpha + \alpha_0) < 1 \quad => \quad 0 < (\alpha + \alpha_0) < 90^\circ$

Value $(\alpha+\alpha_0)$ depends on the shape and direction of motion of the knife, angle of cutting.

The inertia force:

$$P_i = ma = m\, d^2 y(t)/dt^2\ , \tag{4}$$

where m – mass of movable part from cutting mechanism brought to the knife; a – knife acceleration when move in the product.

Establishment of equilibrium equation:

$$m\, d^2 y(t)/dt^2 + F_r + F_{ad} = 0 \tag{5}$$

Consider the most common cases and analyze the model (5) for various structural and mechanical properties of food.

Recognising the strength of adhesion for linearly dependent on the quoted unit to cut surface of cut duration t [2]:

$$P_{ad} = b + at \tag{6}$$

where a and b – experimentally defined coefficients receipt of equation (5). Having in mind the initial conditions $t=0 \quad => \quad y=0, \ \ dy/dt = V_{oy}$:

$$y(t) = V_{oy} t - \frac{t^2 (3F_r + S\cos(\alpha + \alpha_0)(3b + at))}{6m}. \tag{7}$$

Differentiating the equation 7, determine the speed of cutting:

$$\frac{dy}{dt} = \frac{2V_{oy}t - 2F_r t - S\cos(\alpha + \alpha_0)(2bt - at^2)}{2m}. \tag{8}$$

From the equation (8) define the cutting force:

$$F_r = \frac{2V_{oy}t - S\cos(\alpha + \alpha_0)(2bt - at^2)}{2t} - \frac{m}{t}\frac{dy}{dt}\ . \tag{9}$$

The dependence of adhesion strength from reduced to unit area of the cut length of cut t can be nonlinear:

$$m\frac{d^2y(t)}{dt^2}+F_r+Be^{bt}S\cos(\alpha+\alpha_0)=0 . \quad \textbf{(10)}$$

Using a similar previous case activities, we find:

$$F_r=\frac{V_{oy}m}{t}-\frac{BS\cos(\alpha+\alpha_0)(b-be^{bt})}{tb^2}-\frac{dy}{dt}\frac{m}{t} . \quad \textbf{(11)}$$

During cutting of bread, when directly a model that describes the process is known:

$$F_r+G+F_{ad}+P_i=0 \quad \textbf{(12)}$$

where F_r – cutting force;

G - friction force ;

P_i – inertia force;

F_{ad} - adhesion force , in the present case $F_{ad}=0$.

Disclose the members of the equation and solve it:

$$F_r+(C_{mp}+k_1\frac{dy(t)}{dt})+m\frac{d^2y(t)}{dt^2}=0 , \quad \textbf{(13)}$$

$$y(t)=\frac{C_1\cdot m\cdot e^{-\frac{k_1\cdot t}{m}}}{k_1}-\frac{(F_r+C_{mp})t}{k_1}+C_2 , \quad \textbf{(14)}$$

where C_1 and C_2 – are constant.

In the initial conditions t=0 => $y=0$ => $dy/dt=V_{oy}$ there is:

$$y(t)=\frac{(F_r+C_{mp}+V_{oy}\cdot K_1)\cdot m\cdot e^{-\frac{k_1\cdot t}{m}}}{k_1^2}-\frac{(F_r+C_{mp})\cdot t}{k_1}+\frac{(F_r+C_{mp}+V_{oy}K_1)\cdot m}{k_1^2} . \quad \textbf{(15)}$$

Differentiating equation (15):

$$\frac{dy(t)}{dt}=\frac{(F_r+C_{mp}+V_{oy}\cdot K_1)\cdot e^{-\frac{k_1\cdot t}{m}}}{k_1^2}-\frac{F_r+C_{mp}}{k_1} . \quad \textbf{(16)}$$

From the equation (16) the cutting force is defined:

$$F_r=\frac{k_1\frac{dy(t)}{dt}-e^{-\frac{k_1\cdot t}{m}}(C_{mp}+V_{oy}k_1)+C_{mp}}{e^{-\frac{k_1\cdot t}{m}}-1} , \quad \textbf{(17)}$$

where $dy(t)/dt$ - speed of the knife move. It is defined as the average speed between the beginning of the entry of the knife in the product and its outcome.

3. EXPERIMENTAL SYSTEM

Developed experimental set having a simple structure and reliable in operation, which allows to developed mathematical model to determine the cutting resistance. This is a pendulum, the face of the lever which is attached a knife (Fig. 2). The design allows widely vary the speed of the blade and to determine its value.

The knife speed on the entrance or outcome of product is determined by the equation:

$$V=R\sqrt{2\frac{\sum Q_i r_i}{J}(1-\cos\alpha)} , \quad \textbf{(18)}$$

where Q_i – the weight of different details from the leverage mechanism, r_i - distance from the center of gravity of this detail to the axis of pendulum; α - angle from which place or which raises the pendulum (hammer); R – length of the lever; J – moment of inertia of the pendulum for all details.

The obtained equations and developed methodology for analyze enable determining the power of cutting, depending on the speed of the moving blade in the material and other characteristics of the cutting process, to compare the properties of food products to make evaluations of their performance in cutting forces [1].

The results of mathematical and physical modeling are applied to determine rational regimes for food products cutting. For example, we will adduce the results of bread cutting (wheat, 0.5 kg).

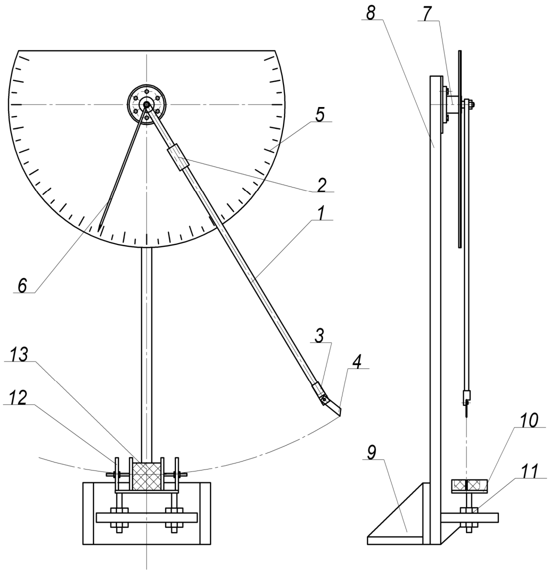

Figure 2: *Scheme of the stand for studying of cutting process:*
1 - lever; 2 - load; 3 – knife holder; 4 -knife; 5 - scale; 6 - indicative arrow; 7 - ax; 8 - column; 9 - base; 10 – working plot; 11 – drive screw; 12 - fixative; 13 - product.

4. RESULTS AND DISCUSSION

Using formula (17) is determined the relative force of cutting (per unit length of the knife) for middle and cortex (Fig. 3 and 4).

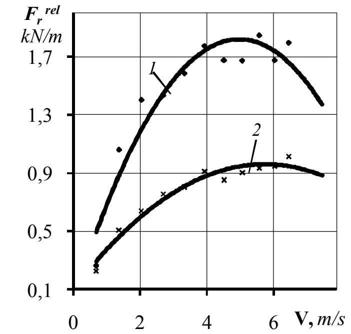

Figure 3: *Influence of the knife speed over the relative cutting effort of the bread core when time for stay of bread,* ***h****:*
*1- **0**; 2-**6**.*

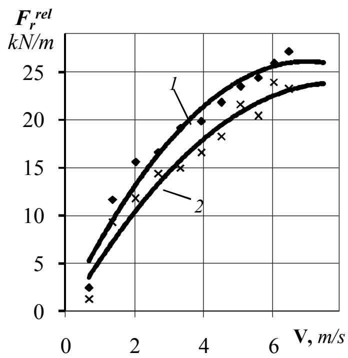

Figure 4: *Influence of the knife speed over the relative cutting effort of the bread cortex when time for preliminary stay of the bread,* ***h****: 1-**0**; 3 – from **6** to **48**.*

The results are set out in range of knife speeds 1-8 m/s.

By increasing the knife speed in the product the cutting force in soft part of the bread rises. At speeds around 5 m/s cutting force is maximum (extreme value for cutting force on the graph), with further increase in speed - decreases. The biggest cutting force in the soft part of the bread - the bread cutting immediately after baking. Similar correlations were obtained for many food products (cheeses, meats, etc).

This is explained as follows: cutting force consists of forces of destruction structural links in the product, forces of the deformation of the product, the force from friction of the knife surfaces. During the cutting process under the edge of the knife arise elastic and then plastic deformation. Plastic deformations, unlike elastic, are developed with a lower speed. At high speeds of deformations the limit stresses and destruction of the product are achieved without significant plastic deformation. However, part of deformation forces are reduced when

cutting the product and the total cutting force is reduced.
The cutting force of the bread crust is constantly growing with increasing of speed (Fig. 4). Destruction without plastic deformation in the cutting process is typical for the most of fragile products.

3. CONCLUSION

It was developed a theory on which base could be modeled a cutting process of food products with different mechanical properties.
It was developed a methodology for experimental studies also, which verify developed theory for cutting of food products.
For the purposes of experimental work was developed a specific experimental apparatus. It allows determining the cutting force, depending on the speed of cutting. Based on conducted experiments with various food products is defined an optimal construction of the knifes.
The optimal structure and mechanical properties of food products, enabling a high quality cuts were defined.
Based on achieved results is recommended bread cutting to be with speed of the knife higher 5 m/s. At the same time cutting force is reduced and plastic deformation in soft core part of the bread under cutting edge of blade, and the product is not deformed when cut.
Feature of development and innovation are made analytical studies and mathematical models development. They make it possible to determine directly the cutting forces for food with different structural and mechanical properties.
Application of the modeling results of the cutting process allows optimizing processes for many food products. In addition to defining rational modes of cutting through the developed methodology can be identified such indicators as the texture of the products where in combination with other indicators makes it possible to assess the quality of the product. This makes it possible to conduct analytical studies in the future and the accumulation of experimental data on the subject

REFERENCES

[1] Gubenia O. O. Usovershenstvovanie processa rezaniia hleba s uchetom vliianiia ego strukturno-mehanicheskih svojstv. Dissertation. (2008), 21 s.
[2] Guc V. S. Adgeziia pishtevh produktov v processah pakovki. Upakovka. № 2 (2006), S. 39-41.ISSN
[3] Zimon A. D. Adgeziia pishtevh mass. Agropromizdat, 1985, p. 272. ISBN