On the uniform convergence of Fourier series to (ψ,β)-derivatives

Elena Radzievskaya
$National University of Food Technologies, Kiev, Ukraine$
radzlena58@gmail.com

In terms of the best approximations of a function in the space Lp, the conditions of existence of its (ψ,β)-derivatives and the uniform convergence of Fourier series to them are determined.

Let L_p be a space of measurable 2π-periodic functions $f(x)$ for which $\int\limits_0^{2\pi} |f(x)|^p\,dx < \infty$, $1 < p < \infty$ and let

$$f \sim \sum_{k=-\infty}^{\infty} \hat{f}_k e^{ikx}$$

be its Fourier series.

Let $\psi(t) > 0$ for $t > 1$ and let β be any fixed real number. If the series

$$\sum_{k=-\infty}^{\infty} \frac{\hat{f}_k}{\psi(|k|)} e^{i(kx+\beta sign k)}$$

is the Fourier series of some summable function, it is called the (ψ,β) - derivatives of a function f and is denoted f_β^ψ. The set of functions that satisfy these conditions is denoted by L_β^ψ.

If $f \in L_\beta^\psi$, and $f_\beta^\psi \in N$ where $N \subset L(0,2\pi)$ we say that the function belongs to the class $L_\beta^\psi N$ [1, c. 142 143].

This report is devoted to the determination of a sufficient conditions of existence of the continuous (ψ,β)-derivative of a function f from L_p and the uniform convergence of the Fourier series of the (ψ,β)-derivative in the terms of the best approximations $E_n(f)_p$. **Theorem.** *Let $\psi(t)$be a positive nonincreasing function which is defined for all $t \geq 1$ and is such that $\psi(2t) \geq c\psi(t)$ for $t \geq 1$ (c is some positive constant) and let the best approximations of the function $f \in L_p$, $1 < p < \infty$ satisfy the condition*

$$\sum_{k=1}^{\infty} \frac{k^{\frac{1}{p}} E_k(f)_p}{\psi(k)k} < \infty.$$

Then, for any real β the function $f \in L_p$ possesses a continuous (ψ, β)-derivative whose Fourier series converges uniformly.

Acknowledgements

1. A. I. Stepanets, Methods of Approximation Theory. I [in Russian], Inst. Math. of the NASU, Kiev (2002),425 p.