BOUNDARY INTEGRAL EQUATIONS FOR PROBLEMS ABOUT PLANE DEFORMATIONS OF LINEAR VISCOELASTIC MEDIUM

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The technological processes of food productions are often related to materials or raw material, mechanical properties of which are viscoelastic. In a plane viscoelastic area $D(t)$, limited by the smooth reserved contour $L(t)$ at $t \geq 0$ integro-differential equation of equilibrium is

$$
\mu \Delta \ddot{u}(\vec{y},t) + (\lambda + \mu) \text{grad} \text{div} \ddot{u}(\vec{y},t) - \mu \int_0^t q(t-\tau)[\Delta \ddot{u}(\vec{y},\tau)] + \frac{1}{3} \text{grad} \text{div} \ddot{u}(\vec{y},\tau) d\tau + \vec{f}(\vec{y},t;\vec{u}) = \vec{0} \quad (1)
$$

at the set tensions $\vec{p}_H(\vec{x},t)$ in contour point $L(t)$. Explanatory notes: $\mu, \lambda$ are instantly-resilient steelg; $\Delta$ is Laplace operator; $\ddot{u}(\vec{y},t)$ is a displacement vector; $\vec{f}(\vec{y},t;\vec{u}) = \rho_0 \vec{m}(\vec{y},t)[1 - \text{div} \ddot{u}(\vec{y},t)]$ is mass force intensity, $\rho_0 = \rho(\vec{y},0)$ is material density, $\vec{y} \in D(t)$; $q(t) = ce^{-\beta t}t^{\alpha-1}$ is Rzhanicyn relaxation kernel ($\beta, \alpha > 0$, $\alpha \in (0, 1)$ are parameters of material); $\vec{n}$ is a normal of the given contour point $\vec{x} \in L(t)$.

The solution of this problem is as a sum of partial solution of equation (1) and viscoelastics potentials of a simple layer: $\ddot{u}(\vec{y},t) = \ddot{u}[\vec{f}] + \sum_{k=1}^{2} e_k l_k \int_0^t \int \vec{v}(l,\tau) \cdot \vec{v}^{(k)}(\vec{y};t-\tau)d\tau dL(\tau)$, (2)

where $\vec{v}^{(k)}(\vec{y};t-\tau)$ is a fundamental solution of equation (1).

The substitution of expression (2) in a boundary condition results in the system of the second type integral equation in relation to a component of the sought vectorial density of potential $\vec{v}(l,t) \in L(t)$:

$$
\pi v_1(l_0,t) + \int_{L(t)=1}^2 \sum_{i=1}^{2} v_i(l,t)K_i(l,l_0;t) \frac{\partial \tilde{\chi}(l,t)}{\partial l} dl + \int_0^t \tilde{k}(t-\tau)d\tau \int_{L(\tau)=1}^2 \sum_{i=1}^{2} v_i(l,\tau)k_i(l,l_0;l,\tau) \frac{\partial \tilde{\chi}(l,\tau)}{\partial l} dl = \psi_1(l_0,t); \quad (3)
$$

$$
\pi v_2(l_0,t) + \int_{L(t)=1}^2 \sum_{i=1}^{2} v_i(l,t)K_i(2l,l_0;t) \frac{\partial \tilde{\chi}(l,t)}{\partial l} dl + \int_0^t \tilde{k}(t-\tau)d\tau \int_{L(\tau)=1}^2 \sum_{i=1}^{2} v_i(l,\tau)k_i(2l,l_0;l,\tau) \frac{\partial \tilde{\chi}(l,\tau)}{\partial l} dl = \psi_2(l_0,t),
$$

where $K_i(l,l_0;t)$ and $k_i(l,l_0;l,\tau)$ are equation kernel; $\tilde{k}(t)$, $\psi_1(l_0,t)$ and $\psi_2(l_0,t)$ are the known functions.

The method of "steps at times" is used for numerical calculations of the proved system of integral equation of the 2-nd type (3).

KEY WORDS Viscoelasticity, relaxation kernel, viscoelastic potential, fundamental solution, potential density, integral equation kernel.