METHODOLOGICAL ASPECTS OF THE MATHEMATICAL THEORY OF FRICTION IT IS APPLICABLE TO BEARING SURFACES OF OVERLOAD DEVICES

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Abstract. *In streaming lines of cargo product containers provided by the application of various design schemes for handling devices. During the execution of operations, for the most part overload loads change the orientation with respect to the original direction of motion. Change the orientation of load on bearing planes carried in difficult cargo plane motion of one or more load-bearing planes simultaneously. To determine the rational parameters of handling devices is to apply mathematical modeling of the movement of cargo. Feature of this simulation is the use of mathematical theory of friction product containers of goods on the surface. As an example of methodology of mathematical simulation of containers of a product operations of processing of a load, results of operations that intends for cargo package in system of four pipelines are considered.*

Keywords: cargo transshipment complex device flat, mathematical modeling, the theory of friction.

I. Introduction

For the enterprises of the food industry, today, the problem of complex automation of final operations of a fabrication cycle remains the actual task, - where the place is taken also by overload and orientation operations. At many food enterprises of Ukraine, highly productive automated lines of a domestic production are successfully used. In product lines on processing of food loads use of different designer decisions on overload operations is provided. Such lines consist not only of pipelines, but also combine operation of a row of auxiliary active and passive working organs [1].

Insufficient illuminance of the questions connected to researches of processes of processing of tare loads in product lines, use of not tested and incomplete equipment leads to excess costs of mounting, connection, energy and materials.

Essential technical equipment downtimes cause origin of jams in overload devices, damage of finished goods.

For the considerable number of the stream and transport systems (STS) of food productions rationally to organize freight handling in places of a joint of single-level and multi-level pipelines.

The type selection of overload and approximate devices depends from: type of tare loads; like transport devices by means of which the overload is carried out; cargo flow, which passes through the overload device; productivity of a process line; ways of relocation of a load are long; admissible overall dimensions of the overload device; observance of safety requirements during implementation of overload operations; economic efficiency.

In case of the organization of execution of operations it is expedient to carry out the comparative analysis of possible constructive options of devices for finding of an effective method of an overload, or orientation in case of the initial parameters.

To avoid possible damage of a tare load it is necessary to analyze characteristic force parameters of process of an overload [2].

Tare loads are in relocation and orientation processes under the influence of the active or passive working organs and devices of machines. In certain cases loads are at the same time under the influence of the active and passive working organs, and thus make difficult plane movement [3]. In the course of interaction of loads with constructive elements of devices and machines, such loadings act them [4]: static, dynamic, shock (are defined by nature of change of loading in time).

Also the gravity and inertia (dispersed on the volume of all load) are considered; surface (depend on the size of the area of contact of a load with constructive elements of technical system).

Change of values of these loadings influences kinematic parameters of relocation, orientation of loads. Interaction of loads with working organs of devices is analyzed on the following parameters: forces and moments of forces of sliding friction; forces of shock interaction; static and dynamic firmness of loads during their movement; interaction responses between a load and elements of working organs.

In case of mathematical simulation of movement of loads in overload devices there are the difficulties connected to the decision of difficult systems of equations, consisting of non-linear differential dependences [2,4,5].

II. Materials and methods

As a mathematical model in case of research of similar operations select system of the mathematical dependences received based on the analysis of operating forces on a load. Specifics of the solution of similar engineering tasks consists in need of step-by-step reviewing of movement of a load for similar overload devices.

Let's analyze difficult plane movement of a load in the horizontal area, the direction of its movement will change thus not only the provision of a load on the bearing plane, but also. In this case the load gets under action of inertial forces, reactive forces (from working surfaces of orienting devices), frictional forces which work in a contact zone between a reference edge of a load and the bearing plane.

For research of operations of an overload and a redirecting of loads on the mobile bearing planes we use the special methodical receptions explained in works [1 … 4,6,7], and also we will take into account some features of difficult plane movement (multistaging and need of search of the additional equations – kinematic or holonomic communications).

The analysis and synthesis of the finite equations gives the chance to receive the complete information about high-quality and quantitative changes of force parameters of movement of loads.

Such constructions (fig. 1) can be the most characteristic examples of devices of an overload at one spatial level [8]:

- the device consists of two pipelines located at an angle 90°, and a rectilinear fixed guide (fig. 1a);
– the device consists of two pipelines located at an angle 90°, and the curvilinear fixed guide (fig. 1*б*);
– the device consists of two pipelines located at an angle 180°, and a rectilinear fixed guide (fig. 1в);
– the device consists of two pipelines located at an angle 180°, (the bearing planes move towards), a circle emphasis (fig. 1г);
- the device for orientation of loads – consists of four horizontal pipelines (fig. 1д).

During research of such operations of an overload it is used such methodological sequence:

• we accept basic data: system configuration, the key geometrical parameters, productivity, standard sizes and mass of a tare load (including physicomechanical properties as tares and the packed-up product);

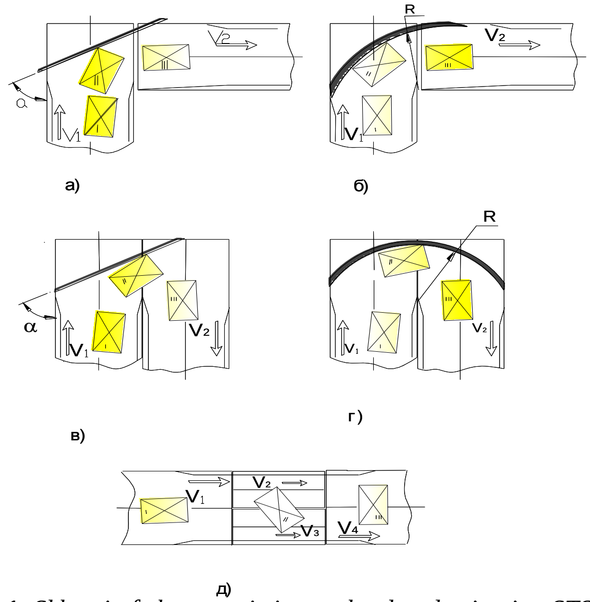

Fig. 1. *Skhemi of characteristic overload and orienting STS devices in which the load makes difficult plane movement on several bearing planes of pipelines.*

- we develop estimated diagrams of characteristic stages of operation of freight handling;
- we define types and number of forces operating on a load in the course of its movement;
- we define the initial and finite living conditions of each stage;
- development of a mathematical model for each stage of movement of a tare load;
- the solution of analytical dependences and assessment of the received parameter values of freight handling.

As an example of implementation of the given methodological sequence we will consider load movement in the device into which composition enter (fig. 2): the giving pipeline 1, two orienting pipelines 3 and 4 and the leading-out pipeline 5. On these pipelines bearing the planes move and at the same time tare loads tare loads 2 concerning the original direction of movement are guided. Important advantage of use of such device is possibility of relocation of tare loads the continuous flow on pipelines 1, 5, and on pipelines 3 and the 4th overload is combined with operation of orientation of a load. Not to create a hindrance in the continuous operation of this system, it is necessary to provide the following ratio of speeds of tapes of giving and orienting pipelines $V_3 > V_1 < V_4$ -, thus the speed of the leading-out pipeline can be coordinated with the speed of the giving pipeline: $V_1 < V_5$. Speeds of orienting pipelines $V_3, V_4 -$ shall be picked up so that the turn of a load 2 on an angle 90° of rather preliminary direction of movement during passing of distance by it was provided l_3.

Making of similar operation is accompanied by difficult plane movement of a load on several bearing planes. For finding of geometrical and kinematic parameters which characterize load movement on the given basic data, we use the theory of movement of a solid body on a rough surface. A little to simplify a mathematical model which describes load movement in the orienting device, we will accept a row of assumptions which won't change significantly physical entity and nature of researched process:

- loads are elasto-plastic bodies, a parallelepiped, are uniformly filled with uniform production and the center of gravity of a load matches its geometrical center;
- values of coefficients of sliding friction of value f_1, f_3, f_4 the unequal : $f_1 < f_3 > f_4$,

where $f_1 -$ friction coefficient between a load and a tape of the giving pipeline 1;

$f_3, f_4 -$ friction coefficients between a load and tapes of orienting pipelines 3 and 4;

Depending on a ratio of force factors which affect a load, and nature of movement of single loads on the bearing planes, we will conditionally provide redirecting operation in the selected system set of such stages:

- first stage – load relocation from the giving pipeline 1 on bearing planes of pipelines 3 and 4 with a speed V_1;
- second stage – load transition with increase in speed from V_1 to V_3 from a tape of the giving pipeline on bearing planes of orienting pipelines 3 and 4;
- third stage – load turn round its center of masses About with simultaneous movement of a load along the OY axis with slip of points of application of equally effective gravity of parts of a load which at the same time are on pipelines 3 and 4;
- fourth stage – load relocation from orienting pipelines 3 and 4 on leading-out pipeline 5 with simultaneous change of speed of a load to V_5.

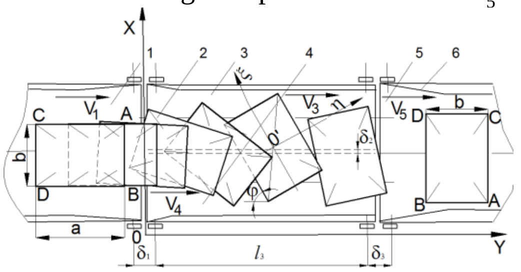

Fig. 2. *The diagram of step-by-step orientation of a tare load in system of four horizontal pipelines.*

Let's in more detail provide researches of the third stage which is characterized by difficult plane movement of a load on two bearing planes of pipelines 3 and 4.

III. Results and discussion

Let's accept that the load center of gravity O' moves rectilinearly on the OY axis, and the load rotates round center O.

Basic data:

$$t_{3n} = 0; \quad f_3 > f_4; \quad V_3 > V_4, \quad L'_3 > L'_4; \quad y_{3n} = 0{,}5a;$$

$$\dot{y}_{3n} = \dot{y}_{2_k}.$$

The diagram of force loading at this stage is provided in fig. 3.

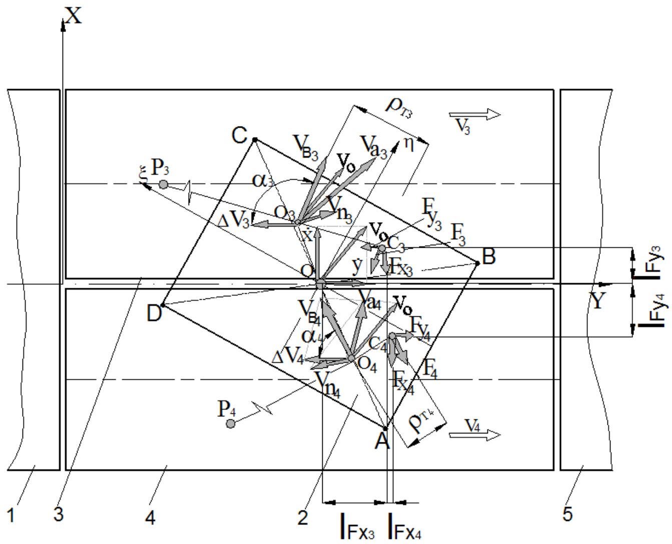

Fig. 3. *The diagram of force loading at the third stage.*

Load motion equation at this stage we will write in a look:

$$\begin{cases} m\ddot{y} = F_{y_3} - F_{y_4}; \\ I\ddot{\varphi} = F_{x_3} l_{F_{x_3}} + F_{y_3} l_{F_{y3}} - F_{x_4} l_{F_{x4}} + F_{y_4} l_{F_{y4}}, \end{cases} \quad (1)$$

as $F_{x_3} = F_{x_4}$, *then* $\ddot{x} = 0$,

then x, y - coordinates of center of masses of a load,

m- mass of a tare load;

φ - angular coordinate of a load;

$F_{x_3}, F_{x_4}, F_{y_3}, F_{y_4} -$ projections to axes X i Y resultant vectors F_3 and F_4 frictional forces which affect a supporting plane of loads from tapes of giving and trunk pipelines, - are determined by formulas:

$$F_{x_3} = F_3 \cdot \sin\alpha_1, F_{x_4} = F_4 \cdot \sin\alpha_2,$$
$$F_{y_3} = F_3 \cdot \cos\alpha_1, F_{y_4} = F_4 \cdot \cos\alpha_2 \quad (2)$$

α_1, α_2 - angles between absolute and relative speeds of parts of a load which are located on pipelines 3 and 4.

$$\alpha_1 = \arcsin\left(\frac{V_{n_3}\cos\beta}{V_{в_3}}\right),$$
$$\alpha_2 = \arcsin\left(\frac{V_{n_4}\cos\beta}{V_{в_4}}\right); \quad (3)$$

$V_{n_3}, V_{n_4} -$ normal components of circumferential speed of center of masses of parts of a load with centers $O_3, O_4.$, are defined as:

$$V_{n_3} = \dot{\varphi} \cdot l_{OO_3},$$
$$l_{OO_3} = (y_{O_3} - y)/\cos\beta = x_{O_3}/\sin\beta \quad (4)$$

$$\beta = arctg(x_{O_3}/y_{O_3} - y), \quad (5)$$

$$V_{n_3} = \dot{\varphi}\frac{y_{O_3} - y}{\cos(arctg(x_{O_3}/y_{O_3} - y))}; \text{ then} \quad (6)$$

$$V_{n_4} = \dot{\varphi} \cdot l_{OO_4}, \quad \text{or}$$

$$V_{n_4} = \dot{\varphi}\frac{y_{O_4} - y}{\cos(arctg(x_{O_4}/y_{O_4} - y))}; \quad (7)$$

$x_{O_3}, x_{O_4}, y_{O_3}, y_{O_4} -$ coordinates of centers of masses of parts of a load which are on orienting pipelines 3 and 4. l_{OO_3}, l_{OO_4} - distances between layout of centers of masses of a load O_3, O_4.

β - angle between l_{OO_4} and coordinate axis of Y. It is necessary to mark, as the areas of parts of a load which are on pipelines 3 and 4, always are equal among themselves (proceeding from the accepted assumptions), $V_{n_3} = V_{n_4}$.

The relative speeds of centers of masses of parts of a load we define from the equations:

$$\Delta V_3 = V_3 - \dot{y},$$
$$\Delta V_4 = V_4 - \dot{y}, \quad (8)$$

Using the cosine law, we find values of the relative speeds $V_{в_3}, V_{в_4}$, that is speeds of points O_3, O_4 in relation to fixed coordinate system.

$$V_{в_3}^{\ 2} = V_{n_3}^{\ 2} + \Delta V_3^{\ 2} - 2V_{n_3}\Delta V_3 \sin\beta, \quad (9)$$

Operating with the equations (6) i (7), we find:

$$V_{в3} = \sqrt{\left(\dot{\varphi}\frac{y_{O3} - y}{\cos\left(arctg\frac{x_{O3}}{(y_{O3} - y)}\right)}\right)^2 + \dots + (V_3 - \dot{y})^2 - 2\dot{\varphi}(V_3 - \dot{y})x_{O3}};$$

$$V_{в4} = \sqrt{\left(\dot{\varphi}\frac{y_{O4} - y}{\cos\left(arctg\frac{x_{O3}}{(y_{O4} - y)}\right)}\right)^2 + \dots + (V_4 - \dot{y})^2 - 2\dot{\varphi}(V_4 - \dot{y})x_{O4}}. \quad (10)$$

Distances from points of application of a projection of forces to center of masses of a load O' are determined by such dependences:

$$l_{F_{x3}} = y - y_{O3} + \rho_{T3}\sin\alpha_1,$$
$$l_{F_{y3}} = x_{O3} + \rho_{T3}\cos\alpha_1,$$
$$l_{F_{x4}} = y + y_{O4} + \rho_{T4}\sin\alpha_2,$$
$$l_{F_{y4}} = -x_{O4} + \rho_{T4}\sin\alpha_2, \quad (11)$$

where: $\rho_{T_{3,4}} -$ radiuses of application of force of sliding friction concerning parts of a load which are located on orienting pipelines 3 and 4, are defined as:

$$\rho_{T_3} = \frac{M_3}{F_3}, \quad \rho_{T_4} = \frac{M_4}{F_4}, \quad (12)$$

where $M_{3,4}$ - the moment of sliding friction of a load on reference surfaces of pipelines 3 and 4.

On approximating dependences we find [2,4]:

$$M_3 = \frac{S_3}{S}L_0(1 - \exp(-k_2 \cdot r_3));$$
$$M_4 = \frac{S_4}{S}L_0(1 - \exp(-k_2 \cdot r_4));$$
$$F_3 = \frac{S_3}{S}mgf_3(1 - \exp(-k_1 r_3 - A_0)),$$
$$F_4 = \frac{S_4}{S}mgf_4(1 - \exp(-k_1 r_4 - A_0)). \quad (13)$$

Where *S* – load bearing surface area; S_3, S_4 – area of parts of a reference surface of the load located on pipelines 3 and 4 respectively; k_1, k_2 – approximating coefficient;

r_3, r_4 – radius distance from center of masses to distance from centers of masses and to the instantaneous rotation centers of a load; L_0 – the moment of friction of a load in case of r=0;

$$A_o = \ln(1 - \frac{F_0}{mgf_{3(4)}},$$

F_0- values of the principal vector of frictional forces is defined in case of r=0.

Coordinates of centers of masses of parts of a load in a case when the load completely moved on orienting pipelines 3, 4, can be found on dependences:

$$\begin{aligned} x_{0_3} &= \xi_{0_3}\sin\varphi + \eta_{0_3}\cdot\cos\varphi,\\ x_{04} &= -\xi_{04}\sin\varphi - \eta_{0_4}\cdot\cos\varphi,\\ y_{0_3} &= y - \xi_{0_3}\cos\varphi + \eta_{0_3}\cdot\sin\varphi,\\ y_{04} &= y + \xi_{04}\cos\varphi - \eta_{04}\cdot\sin\varphi. \end{aligned} \qquad (14)$$

Where ξ_{03}, η_{03}, ξ_{04}, η_{04} – coordinates of centers of masses of parts of a load located on pipelines 3 and 4 in system ξ and η.

The third stage will end provided that the load will move on the pipeline 5, и $\gamma = arctg\frac{a}{b}$; *а, в* – geometrical parameters of a reference surface of a tare load. In case of execution of this condition there will come the fourth stage which also is characterized by difficult plane movement until condition execution $L_3' \le L_4' + L_5'$,where L_3', L_4' L_5' - the moments of frictional forces of parts of a load which are on pipelines 3,4,5.

Having received necessary dependences and basic data, it is possible to decide system of equations relatively y and φ by means of an application-oriented packet MathCAD.

Results of numerical calculations of analytical dependences which describe load movement at all stages in the form of diagrams (fig. 4). Analyzing them, it is possible to claim that it is possible to control operation of orientation of a tare load at the expense of change of values of the key geometrical, physical and kinematic parameters of the most overload system.

Together with it is possible to recommend that during carrying out calculations, it is long orienting pipelines would allow load turn on φ=90º.

Also, it is necessary notify a situation of turn of a load on a bigger angle, for this purpose in overload system additional guides on transition to the pipeline 5 are set. Therefore the transition mode will be characterized by speed V_5.

IV. Conclusions

1. The carried-out analysis of force interaction of a load with working organs of overload devices gives the chance to set that the main external loads of a tare load is: forces and moments of frictional forces; responses of forces from load interaction with surfaces of working organs and inertial forces.

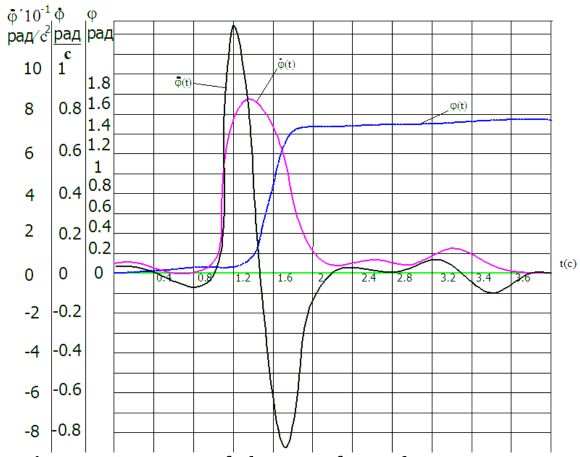

Fig. 4. *Diagrams of change of angular parameters of movement груа during operation of its redirecting on 90 ° rather original movement in overload system from four pipelines.*

2. Characteristic restraining factor of increase of productivity of overload devices is restriction of force interaction on tare loads and therefore for support of the given rhythm of production it is necessary to provide them is constructive for minimization of efforts.

3. The executed analytical researches of operation of an overload of tare loads in system of four pipelines use the mathematical theory of friction between a load and supporting planes of pipelines. Also prove possibility of intensification of process of a redirecting when processing big freght traffics.

References

[1] Gavva O.M., Kryvopljas-Volodina L.O. Perevantazhennja vantazhiv u potokovo-transportnyh systemah odnogo rivnja //- Naukovi praci NUHT, 2004, № 2 – S.17-21

[2] Krivopljas A.P. Razrabotka teoreticheskih osnov peremeshhenija shtuchnyh gruzov v potochnyh linijah ukrupnenija gruzovyh edinic pishhevyh proizvodstv i ih prakticheskoe prilozhenie.- Dis. … dokt. tehn. nauk..- K., 1988.- 500s.

[3] Krivopljas A.P. Paketoformirujushhie mashiny/ Krivopljas A.P., Kukibnyj A.A., Bespal'ko A.P., Burov A.A.. – M.: Mashinostroenie,1982. – 239s.

[4] Gavva O.M. Naukovi osnovy rozrahunku parametriv potokovo-transportnyh system harchovyh vyrobnyctv .- Dys. dokt. tehn. nauk. - Kyev, 1996.- 376s.

[5] Ivanovskij K.E. Peregruzochnye ustrojstva konvejerov shtuchnyh gruzov/ Ivanovskij K.E., Obolenskij A.S. – M.: Mashinostroenie, 1966.- 208s.

[6] Dashhenko A.I. Proektirovanie avtomaticheskih linij /Dashhenko A.I., Belousov A.P.: M.:Vyssh. shkola, 1983.-328s.