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SELECTION OF THE STABILITY REGION OF LINEAR DYNAMIC SYSTEMS WITH FRACTIONAL ORDER CONTROLLERS

From the beginning of the development of the theory of integro-differential calculus of fractional order, its first applications in control problems appeared only about 50 years ago. It has been shown that fractional calculus becomes an effective tool for describing numerous dynamic systems. The classical results of the *PID* control theory have spread to the fractional order controllers, which denote how $PI^{\lambda}D^{\mu}$, where λ and μ are the orders of integration and differentiation of the error signal, with orders λ and μ may have valid non-integer (fractional) values.

Analytical studies and computational experiment in the MATLAB environment have been conducted.

The well-known problem of the allocation of the global region of stability (*D*-split method) required the distribution of fractional dynamic systems in the space of the parameters of the adjustment $PI^{\lambda}D^{\mu}$-regulator, depending on the value of the orders of powers λ and μ.

The purpose of the article is to study the possibility of applying the *D*-split method to automatic control systems for process control with fractional controllers.

A fundamental operator ${}_aD_t^{\gamma}$ is often referred to as a differintegrator.

$$ {}_aD_t^{\gamma} = \begin{cases} d^{\gamma}/dt^{\gamma}, & \gamma > 0, \\ 1, & \gamma = 0, \\ \int_a^t (d\tau)^{-\gamma}, & \gamma < 0, \end{cases} \tag{1} $$

where γ – fractional order, a –constant associated with the initial conditions.

More fundamental is the definition of Grunwald-Letnikov for the order γ according to which

$$ {}_aD_t^{\gamma} f(t) = \lim_{h\to 0} \frac{1}{h^{\gamma}} \sum_{j=0}^{[(t-a)/h]} (-1)^j \binom{\gamma}{j} f(t-jh), \tag{2} $$

where $\binom{\gamma}{j} = \frac{\Gamma(\gamma+1)}{\Gamma(j+1)\Gamma(\gamma-j+1)}$, $\Gamma(x)$ – gamma Euler's function, $h > 0$ – gain of the time coordinate, $f(x)$ – the function to which the operator of the differential integration is used, $[\cdot]$ – means an integer part of the number. This definition shows that integer derivatives require the use of finite series, and fractional derivatives – an infinite number of members of a series.

It can be proved that the Laplace transform, which is the basis of the definition of the concept of a transfer function, for the differintegrator has the form

$$ L\left\{{}_0D_t^{\gamma} f(t)\right\} = \int_0^{\infty} e^{-st}\, {}_0D_t^{\gamma} f(t)dt = s^{\gamma}F(s) - \sum_{j=0}^{n-1} s^j (-1)^j\, {}_0D_t^{\gamma-j-1} f(t)\Big|_{t=0}, \tag{3} $$

where $F(s) = L\{f(t)\}$ – ordinary Laplace transform function $f(x)$, n – an integer that satisfies the condition $n-1 < \gamma \le n$. Note that if ${}_0D_t^{\gamma-j-1} f(t)\Big|_{t=0} = 0$, $j = 0,1,2,...,n-1$, then from (3) it follows that $L\left\{{}_0D_t^{\gamma} f(t)\right\} = s^{\gamma}F(s)$. Systems with fractional orders have transfer functions of arbitrary real order.

Consider the transfer function of fractional order, which is given by the following expression

$$G(s)=\frac{N(s)}{D(s)}=\frac{b_n s^{\beta_n}+b_{n-1}s^{\beta_{n-1}}+\ldots+b_1 s^{\beta_1}+b_0 s^{\beta_0}}{a_n s^{\alpha_n}+a_{n-1}s^{\alpha_{n-1}}+\ldots+a_1 s^{\alpha_1}+a_0 s^{\alpha_0}}=\frac{\sum_{i=0}^{n} b_i s^{\beta_i}}{\sum_{i=0}^{n} a_i s^{\alpha_i}}, \tag{4}$$

where a_i, b_i, $\beta_n > \beta_{n-1} > \ldots > \beta_1 > \beta_0 \ge 0$, $\alpha_n > \alpha_{n-1} > \ldots > \alpha_1 > \alpha_0 \ge 0$ – arbitrary valid numbers.

In the time domain, the transfer function corresponds to an inhomogeneous differential equation of the fractional order of the form

$$\sum_{i=0}^{n} a_i D^{\alpha_i} y(t)=\sum_{i=0}^{n} b_i D^{\beta_i} u(t), \tag{5}$$

where $y(t)$ – exit, and $u(t)$ – input of the control object, ${}_a D_t^{\gamma}$ – differintegrator.

In the general structure of the closed control system of fractional order with one input and one output is presented $y(t)$ – output, $r(t)$ – input request signal, $e(t)$ – error (mismatch), $u(t)$ – control signal, $G(s)$ – transfer function of the control object, $C(s)$ – transfer function of the fractional order controller.

Transmission function of the fractional $PI^{\lambda}D^{\mu}$ -controller has the form

$$C(s)=k_p+k_i s^{-\lambda}+k_d s^{\mu}, \tag{6}$$

where λ и μ – fractional orders whose values belong to the region $(0,2)$, k_p, k_i, k_d – adjusting parameters of the regulator.

To select the region of stable system stabilization (control object with the controller) we use the *D*-split method, the parameters space.

Recall that according to this method, the boundary between the areas of stability and instability in the space of the configuration parameters is three parts: $\Gamma=\Gamma_0+\Gamma_\omega+\Gamma_\infty$. Constituent Γ_0 is determined from the condition of intersection of the real root of the characteristic equation of the imaginary axis s - plane with $s=0$. That is, the component Γ_0 is found by way of substitution $s=0$ in the equation $P(s)=0$, where $P(s)$ which is determined by the equation. It follows that Γ_0 can be determined from the condition $p_0=0$, if the value of the smallest order q_0 equals 0, i.e. with $s^{q_0}=1$. If $q_0\ne 0$, i.e. $s^{q_0}\ne 1$, then the boundaries Γ_0 does not exist.

On the basis of the D-split method, analytical expressions were obtained, which describe the boundaries of the global region of stability of linear dynamic systems of fractional order of type "input-output" with fractional $PI^{\lambda}D^{\mu}$ -regulators. The stability areas are built on the basis of computational experiments in the space of the parameter settings for fractional $PI^{\lambda}D^{\mu}$ - regulators for fixed orders of diereintegrators in the regulator. An appropriate algorithmic software is developed.