On uniform convergence Fourier series

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In terms of the best approximations of a function in the space L_p was found the condition of existence (ψ,β)-derivatives of the function belonging to L_q when $1 \le p \le q \le \infty$.Also, a condition was found when this derivative is continuous and its Fourier series uniformly converges to it.

Let L_p be a space of measurable 2π-periodic functions $f(x)$, and $\int\limits_0^{2\pi} |f(x)|^p\, dx < \infty$, $1 < p < \infty$; $E_n(f)_p$ be the best approximation of the function $f(x)$ in the metric of L_p by trigonometric polynomials of the order of at most $n-1$ and $\omega_k(f,\delta)_p$ be the modulus of smoothness of the k-th order (k is a natural number) in the space $L_p(0,2\pi)$. According [1] we denote through $f^{\psi}_{\beta}(x)$ a (ψ,β)-derivative of a function f.

The following statement has been proved.

Theorem 1. Let $\psi(t)$ be such positive nonincreasing function which is defined for all $t \ge 1$ such that $\psi(2t) \ge c\psi(t)$ (c is some positive constant), and let the best approximations of the function $f \in L_p$, $1 \le p \le q \le \infty$ satisfy the condition

$$\sum_{k=1}^{\infty} k^{\frac{1}{p}-\frac{1}{q}-1} \frac{E_k(f)_p}{\psi(k)} < \infty.$$

Then the function f has the (ψ,β)-derivative which belongs to L_q , and

$$\sum_{k=1}^{\infty} k^{\frac{1}{p}-\frac{1}{q}-1} \omega_p(f^{\psi}_{\beta}, \frac{1}{k}) < \infty.$$

Corollary. Let the best approximations of the function $f \in L_p$ satisfy the condition

$$\sum_{k=1}^{\infty} k^{\frac{1}{p}-1} \frac{E_k(f)_p}{\psi(k)} < \infty.$$

Then the function f has a continuous (ψ,β)-derivative and its Fourier series converges uniformly to it.

[1]Stepanets A. I. *Methods of Approximation Theory. I* [in Russian], Inst. Math. of the NASU, Kiev (2002).

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