MATHEMATICAL MODELING OF DEFORMATION DURING THE FLAT CARDBOARD CREASING

Poliezhaiev[1] I., A. Gavva[2]

[1] Department of Printing Machines and Automated Complexes, Institute of Publishing and Printing, National Technical University of Ukraine "Kyiv Politechnic Institute", 37, Prospect Peremohy, Kyiv, Ukraine, e-mail: ivan.poliezhaiev@gmail.com
[2]Chair of technical mechanics and packing technique, Faculty of engineering mechanics and packing technique, National University of Food Technologies, Volodimirs`ka St., 68 - Kiev, Ukraine

Abstract: *The relevance and the way of flat paperboard creasing mathematical modeling with the finite element method are described in the article. The quality of creasing is influenced by many factors, the main ones are the physical and mechanical properties and structure of paperboard, shape and size of creasing tools and parameters of creasing matrix. Mathematical model of creasing allows predicting the influence of these factors on the process of creasing and quality, and selecting rational parameters.*

Keywords: creasing, cardboard, paperboard deformation, paperboard folding.

I. Introduction

The need to compliance with the optimal values of the load in the contact zone of cardboard with abundant running tools and prevent damage to the material during creasing justifies the importance and relevance of research.

The purpose of the study is to determine the rational parameters of creasing a flat paperboard using mathematical modeling.

II. Materials and methods

The main properties of cardboard are: appearance and performance characteristics. The exterior is characterized by suitability for printing, laundry, tidying up paint and resistance to abrasion. Operating properties are associated with the physical and mechanical properties of cardboard. These properties relate to how the board will withstand the surrounding factors. [1]

Cardboard has a linear elastic behavior to a given boundary - Yield. This means that the force applied to the cardboard is proportional to the deformation caused by the applied force. If the action of force stops, cardboard restores its original size. This is summarized in Hooke's law described below.

Cardboard is deformed beyond the elastic showing the elastic-plastic properties. This means that the applied force is no longer proportional to strain, see Figure 1. When the action force stops cardboard does not restore its original size. The value of the elastic limit is usually 0.2% elongation. [2]

Feature fibers and cardboard production determine the physical and mechanical properties of the material, which can be regarded as very close to orthotropic.

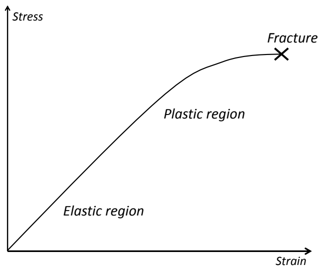

Figure 1. *The elastic and plastic properties of a typical paperboard*

This means that the materials will have different properties in three orthogonal directions: paperboard fiber direction coincides with the longitudinal (machine direction) - MD, direction perpendicular to the fiber paperboard machine running - CD and ZD - thickness direction, as shown in Figure 2.

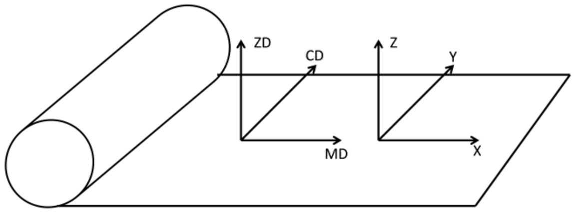

Figure 2. *The main directions of stress perception cardboard*

To ensure correct and line quality and reduce bending force to be put to bending, cardboard slabs are subjected to preliminary processing - creasing. [3]

The process of creasing - a preliminary application to the material through a bend in the form of grooves particular profile is the process of pulling out local material. Creasing is used to reduce the stiffness of sheet cardboard pieces on line bending, significant facilitation of fold formation and improve quality parameters of cartons, especially in its forming machines automatic action.

Creasing can be divided into three stages. In the first phase cardboard blank, when lowering blanking dies, elastic fixed position presses on counter-matrixes. In the second stage creasing tool (with the standard, narrowed or thickened molding head) draws a blank cardboard into two axis directions. In the third stage there is a compression board under creasing tool and counter-matrixes.

Creasing tools put pressure on the work piece so that the board is deformed by just buried matrix elements, with the formation of sharp lines creasing. The macro graphic of creasing process is shown in Figure 3. [4]

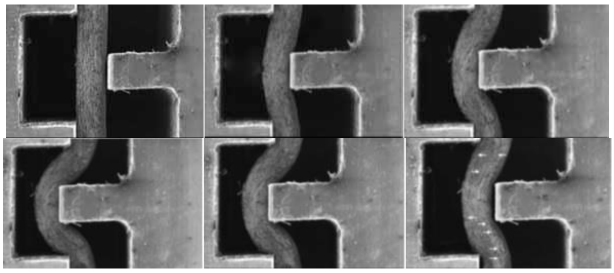

Figure 3. *Macroscopic image of creasing process.*

When creasing, fibrous connections between layers of cardboard are destroyed and damaged, fibers of plastic deformation occurs in cardboard layers. There are creasing high strain shear and compression in the area. Layering shift is caused by layers of cardboard, which reduces its bending stiffness in further assembly. Thanks to local damage in the area, creasing cardboard ideally behaves like a hinge that will improve the quality and productivity of the process assembly. Thus, after creasing the quality can be judged by the ease of folding, i.e. assembly. To assess the quality of creasing, you need to consider many factors that affect it, such as moisture cardboard, width of creasing groove, depth of penetration tools and so on. However, it has been experimentally found that the most significant factor is the thickness of the cardboard. Thicker slabs require wider range and groove. Therefore, they are less sensitive to inaccurate case between the ruler and the groove.

Macroscopic image running them and drawn samples are shown in Figure 4. Based on these images, two important conclusions can be taken into account when constructing mechanical models of creasing.

Firstly, plastic deformation occurs in the area of creasing. Plastic deformation projects thickness of cardboard, ZD, starts after creasing 5% below the creasing line. A special form of creasing zones, after interaction with the tool is caused not only by plastic deformation in the plane ZD, but also by stress and shear stress of the outer layer and stretching towards MD.

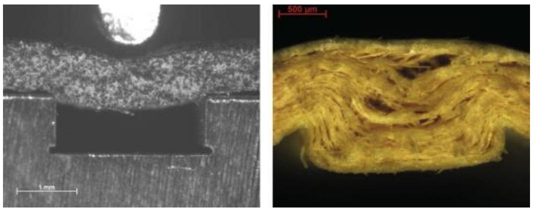

Figure 4. *Macroscopic image of cardboard blanks after creasing (left) and folding (right)*

Secondly, microscopic images are obtained after assembly lines are running to show that the inner layer of cardboard was separated from the outer layers - especially from the upper layer. The very inner layer is divided into several layers of paper. The lower layers were bent and curved inward, as shown in Figure 4 b. [5]

The number of such layers is about eight, without taking into account a number of ultrafine. This behavior detachment is a natural phenomenon for cartons, although the number of layers formed may vary.

During the creasing in the area between the tools and running the creasing groove matrix, the coplanar tension compression and shear stresses are not coplanar. In the industrial creasing (as it is shown in Fig. 5), in the field next to the creasing area, there is the tension in the plane of the work piece. [6]

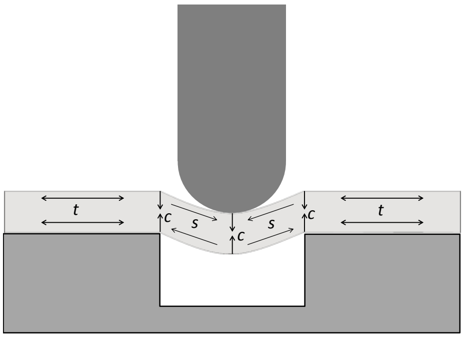

Figure 5. *Diagram of the deformation and stress state in the creasing zone during creasing: t - tension, c – compression, s - shear.*

Not outside of the bend formed tension, but inside it – there is a compression to create a rounded bottom layer. Moreover, the ability to flake board is a very important property for its assembly. Layering is a mode of destruction of paper or paperboard, where the surface is refracted parallel to the plane of the sheet. This type of damage takes place in case of a positive cardboard folding mechanism since it can reduce stress on the outside of the bend, which will enable to reduce the risk of cracks on the outside of the bend. Compilation of cardboard is largely based on the workpiece ability to the internal stratification to compression stresses occurring inside the bend formed by the inner relief. When the separation takes place, stress arising in place of the workpiece, usually 50 to 100 times smaller than the tension arising in the preparation of blanks without prior creasing. [7]

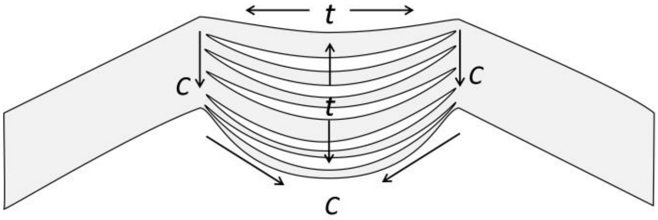

Figure 6. *Diagram of the deformation and stress state in the creasing zone during folding: t - tension, c – compression.*

During the drafting board, its inner layers are easily bending because they have curved shape and are in a compressed state (Fig. 6). As the thin layers in the area of creasing are easier to bend than cardboard outside creasing, bending occurs only along the lines of creasing. However, some non-planar tension may arise within the fold through fibers that took off the surfaces and are in the extended position. The outer layers, including the top layer are under tensile stress, which can lead to cracking and fracture of the upper layer of the workpiece. Thus, creasing and folding must be done so that the tension in the plane of the upper layer is minimized.

Accurate modeling of creasing lines behavior is a key factor in the development of models and approximate mathematical modeling process of creating cardboard packaging.

To predict the behavior of cardboard under the influence of various parameters of creasing we have created a mathematical model. This model should be constructed in such a way that it could be relatively easily implemented in a computer program using the finite element method. To simulate the cardboard we used a holistic model that describes the behavior of cardboard material combined with stratification model to explain the formation of different layers of paper. The most important elements of the mathematical model are elastic-plastic composite materials.

Needless to say, the process of creasing results in lower elasticity, rigidity and strength of cardboard. This deterioration is due to the combined effect of separation, the width W field and creasing cardboard thickness δ. The geometric effect is achieved by creasing permanent deflection of cardboard layers. Depending on the intensity j, creasing, more or less significant changes in cardboard are observed after creasing, which reduces the stability of compressed layers bending.

Based on the observation of physical processes occurring at creasing, it provides:

- The strength and stiffness for reduced crease line of creasing, creasing process in proportion to the intensity measured by the initial nominal shear strain γ, defined in equation (1);

$$\gamma = \frac{2h}{W}$$

(1) where h- value recess creasing tool, W - width of the groove.

- Increased depth penetration (above γ) of creasing tool has got two main consequences: is a bundle of thinner layers of cardboard; remains more out of plane deflection (Figure 7.)

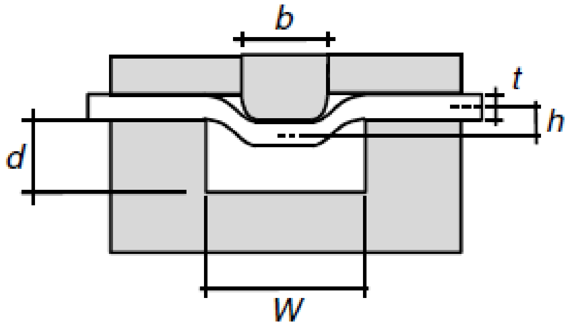

Figure 7. *Geometrical parameters of penetration creasing tool,*
where h - depth of penetration creasing tool and W is the width of the groove, b - width of creasing tool, d - depth of the groove, t - thickness of the paperboard.

- Deterioration of mechanical properties of cardboard in place of creasing is the result of reducing the thickness and the permanent deflection stratified elements. [8]

The finite element method (FEM) (its practical application is often known as finite element analysis (FEA)) is a numerical technique for finding approximate solutions of partial differential equations (PDE) as well as integral equations. The solution approach is based either on eliminating the differential equation completely (steady state problems), or rendering the PDE into an approximating system of ordinary differential equations, which are then numerically integrated using standard techniques such as Euler's method.

In solving partial differential equations, the primary challenge is to create an equation that approximates the equation to be studied, and is numerically stable, meaning that errors in the input and intermediate calculations do not accumulate and cause the resulting output to be meaningless. The finite element method is a good choice for solving partial differential equations over complicated domains [3].

FEM uses a complex system of points called nodes forming elements which make a grid called a mesh. The elements of the mesh are programmed to contain the material and structural properties, which define how the structure will react to certain loading conditions. Nodes are assigned at a certain density throughout the material depending on the anticipated levels of stress of a particular area and they transfer the stress from element to element. Points of interest may consist of: fracture points, fillets, corners, complex details, high stress areas, etc.

The finite element method is originated from the need for solving complex elasticity and structural analysis problems in civil and aeronautical engineering. Its development can be traced back to the work by Hrennikoff. While the approaches used by these pioneers are different, they share one essential characteristic: mesh discretization of a continuous domain into a set of discrete sub-domains, is usually called elements. Starting in 1947, Zienkiewicz from Imperial College gathered those methods together into what would be called the Finite Element Method, building the pioneering mathematical formalism of the method. [2]

III. Results and discussion

The equations of elastic-plasticity in traditional tensor notations are as follows:

$$\begin{cases} \partial_t \sigma = E : (\partial_t \varepsilon - \partial_t \varepsilon_p) \\ \partial_t \varepsilon = (\partial_x v)_s \\ \partial_t \varepsilon_p = H(\|\sigma\| - \sigma_s)\lambda(\sigma, \varepsilon) : \partial_t \varepsilon \\ \rho \partial_t v = \partial_x \sigma \end{cases} \qquad (2)$$

Experimental data show the presence of the diagrams strain softening paperboard areas where there is a voltage drop with increasing deformation. Through these softening initial boundaries value problems are incorrect (broken Hadamard criterion):

$$\partial_t \sigma : \partial_t \varepsilon > 0 \qquad (3)$$

This test is called the stability criterion of material or Drakker criterion. Thus, increasing damage to the elastic properties of the material (elastic modulus and yield strength) is reduced to zero, which means complete destruction of structured material.

To simulate the process of creasing system board there must be equations of the theory of elastic-plastic deformation strength for small supplement kinetic equation for the initial nominal shear strain and elastic modulus and the dependence of the yield strength of the original nominal shear strain. This system of equations will look like:

$$\begin{cases} \rho \partial^2 U = \nabla \sigma \\ \sigma = E(\gamma) : (\varepsilon - \varepsilon_p) \\ \partial_t \varepsilon_p = \lambda_p \dfrac{\partial F_p}{\partial \sigma} H(F_p) H(\sigma : \partial_t \varepsilon) \\ \partial_t \gamma = H(F_d)\Gamma(\varepsilon, \varepsilon_p, \gamma) + r_\gamma \\ \varepsilon = 1/2(\nabla \otimes U + (\nabla \otimes U)^T) \\ F_p(\sigma) = 3/2(\sigma' : \sigma')/\sigma_p^2 - 1 \\ F_d = F_d(\varepsilon, \varepsilon_p, \gamma) \end{cases} \qquad (4)$$

where ρ - density of paperboard; U - displacement vector creasing tool; σ - stress tensor; $\sigma' = \sigma$ - (σ :I)I/3 - deviator stress; $E(\gamma)$ - tensor of elasticity modules, which is a function of the initial nominal shear strain γ; ε - strain tensor paperboard; ε_p - plastic deformation of tensor paperboard; λ_p - coefficient of law of plastic yield stress, defined by plasticity: $F_p(\sigma) = 0$,Symbol «$\otimes$» contains external tensor product; F_p - load function; H - Heaviside’s function equals to zero for negative values of the argument and ones otherwise; σ_p - limit of elasticity paperboard; I - tensor unit; F_d - function by provided destruction inherent value which allows the accumulation of initial nominal shear strain; r_γ - not thermomechanical source of primary shear strain, Symbol «:» contains double scalar product of tensors.

The initial conditions that complement this system of equations will have the form:

$$\begin{aligned} &U\big|_{t=0} = 0 \\ &\partial_t U\big|_{t=0} = 0 \\ &\varepsilon_p\big|_{t=0} = 0 \\ &\gamma\big|_{t=0} = 0 \end{aligned} \qquad (5)$$

In order to solve this problem it is necessary to supplement the main (kinematic) or natural (dynamic) boundary conditions. Boundary conditions depend on the specific task and are critical to address it.

IV. Conclusions

Finite Element Modeling using a system of equations of the theory of elastic-plastic deformation strength for small considering the kinetic equation for the initial nominal shear strain and elastic modulus and the yield strength dependence on the original nominal shear strain, will determine the optimal parameters for quality creasing process, providing certain key and natural boundary conditions.

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