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Fractal analysis of distillation unit time series in prediction and control problems

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Abstract

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Introduction. The behavior of distillation unit as a control object is characterized by stochastic and chaotic manifestations. This requires their identification by the nonlinear dynamics methods for organizing the specific respective control strategies.

Materials and methods. We used the synergetic methods and the theory of deterministic chaos for solving problems of distillation unit prediction and control. Time series analysis was conducted using the Rescaled Range algorithm of the analysis of Hurst, the coating method and the correlation methods. Time series are processed using the software package FRACTAN.

Results and discussion. The research of time series of distillation unit showed that distillation processes are characterized by the presence both stochastic and chaotic regimes, the dimension of attractors of which lies in the range of 3 to 8. Most of these series are fractals, that is, despite the significant instability of the process, their behavior remains the same, which makes it possible to predict change of their condition in the future. Correlation entropy index pointed to a time on which we can predict the behavior of our system. Changing regimes of distillation unit functioning is passing with various periodicity, that reaches from 4 to 10 hours.

For the analysis of chaotic state of the process in behavior of the object we used correlation dimension parameter, which showed that this object have significant trend stability (persistence), that characterized by high value of Hurst exponent in the range of 0.7 to 0.96. The predictability of distillation processes behavior is high, especially for the temperature values. It is increasing with the augmentation of attractor fractal dimension and reaching tens of minutes.

If fractal dimension is less than 1.4, then one or more of the forces affects system, which drives the system in one direction. If the dimension is about 1.5, the forces acting on the system are multidirectional, but more or less compensate each other. If the fractal dimension is much more than 1.6, the system becomes unstable and ready to move to a new state. The analysis of fractal dimension of time series of the distillation column bottom pressure showed that fractal dimension is in the range from 1.0 to 1.4, which in turn indicates that the system affects several forces that move it in one direction, that is, the system is stable.

Conclusions. The researched features of distillation unit functioning as the sophisticated nonlinear control object make it possible the realization of resource-saving control strategies based on diagnostic of its behavior by the methods of fractal analysis.

Introduction

Technological complexes of food industries, including distillation unit of the distillery are complex control objects with the characteristics of complex control systems: a high degree of uncertainty of various forms, significant noise, multicriteriality, nonlinear behavior. Decision-making on control in such circumstances is extremely difficult and time-consuming process.

In order to increase the efficiency of complicated technological complex control systems of food production is necessary to provide operational analysis of the information obtained in the operation of control systems. Thus, the development and implementation of new algorithms and models using modern methods of information analysis is actual scientific and technical problem.

The aim of the research is to improve the efficiency of alcohol production by implementing operative analytical processing of data using methods of fractal analysis, that is investigations of time series of observations using such fractal characteristics as the Hurst exponent, the fractal dimension, the correlation entropy and etc., which make it possible to assess how predictable is the control object, to detect precursors of future disasters and states of emergency of technological process and to prevent undesirable states in future.

The distillation processes on the distillery factories is complicated technological complex. Its behavior characterized by the intermittence, that is rotation of modes of stochasticity, randomness and determinism as a result of appearance of spacetime dissipative structures far from thermodynamic equilibrium in the nonlinear field at critical parameter values at the bifurcation points [1,2]. Dissipative spacetime structures are highly ordered formations because of self-organization through the exchange of energy and substance with the environment. They have a certain shape, size and characterized by resistance to small perturbations (fluctuations) [3]. The important characteristics of dissipative spacetime structures are the time of existence, the space of localization and the fractal dimension [4]. Each type of dissipative structures requires the application of specific topologically coordinated control strategies of resonance character. This provides the efficient usage of energetic and material resources. Hence follows the importance of rapid determination features of the behavior, such as type of behavior and its peculiar properties, of the distillation unit as a control object in real time scale.

The problem of the control of a technical objects condition during their functioning is the most acute became because of the transition from the concept of "scheduled and preventive maintenance" to the concept of "condition maintenance." One of the main tasks scientists face today is a necessity of a constant objective monitoring of the current state of the object and, as a consequence, strengthening requirements to monitoring and diagnostics systems [5]. Currently, the main source of information about the state of diagnostic objects is the time series of observations. Then, on this basis, are determined different process characteristics (the spectral, the correlation, the probability) [6].

These series are usually generated by sophisticated nonlinear systems, the description of which in the form of differential equations or discrete mappings are often associated with large difficulties. However, it was found, that these series are usually fractals [6]. In other words, accurate to the scale factor, such series at different scales look like approximately equally. Such a widely spread of the fractal structure is connected with the fact that in reality any irregular processes seek to find self-similar fractal structure as the most energetically favorable [7]. This means, that the nature of their behavior remains the same at all scales, down to a minimum despite the extreme irregularity. Experimentally proved that time series of data, obtained in the research of these systems, are characterized by

fractal properties (the self-similarity, the self-affinity, the fractal dimension). It makes possible to predict their dynamics, to identify hidden correlations, cycles and so on [8].

Thus, for objective reliable analysis of distillation unit as the diagnostic object by fractal time series you need use of adequate algorithms, that is fractal processing algorithms of informational signals. Today, due to the development of the stochastic fractals theory, such a characteristic of time series as an exponent of Hurst H is becoming more popular.

Analysis of different research methods

The Hurst exponent and the fractal dimension of time series determined by the method of H.E. Hurst, who experimentally found that for many of time series is true an expression [9]:

$$\frac{R}{S} = \left(\frac{N}{2}\right)^H \quad (1)$$

$$S = \sqrt{\frac{1}{N} \sum_{i=1}^N (x_i - x_{cp})^2}, \quad (2)$$

where x_{cp} – the arithmetic mean value

$$x_{cp} = \frac{1}{N} \sum_{i=1}^N x_i \quad (3)$$

$$R = \max_{1 \leq u \leq N} (Z_u) - \min_{1 \leq u \leq N} (Z_u), \quad (4)$$

where Z_u – the accumulated deviation of series from the mean x_{cp}

$$Z_u = \sum_{i=1}^u (x_i - x_{cp}) \quad (5)$$

where H – Hurst exponent; R – the calculated in certain way "rescaled range" of the corresponding time series; N – the number of observation periods;

S – the mean square deviation of a series of observations X ; a – a defined constant $(0.5 \div \frac{\pi}{2})$.

Thus, the Hurst exponent determines the normalized factor of amplitude (Rescaled Range) R/S of time series. Hurst exponent is associated with traditional "cell" fractal dimension D by simple correlation [10]:

$$D = 2 - H \quad (6)$$

An important indicator for the analysis is indicator of the correlation entropy, which shows the degree of divergence of close phase trajectories and allows us to estimate the amount of information required to forecast the future behavior of the object. So this coefficient shows the time on which behavior of the dynamic system can be predicted [11].

We used one of the ways of research the fractal structure of time series – calculation of their fractal dimension across the cell dimension D_2 [8].

We used test, which used in practice, to check the chaotic component in the researched time series lies in determining properties of the correlation amount $C_m(r)$ and behavior

correlation dimension $D_m(r)$ depending on the dimension of attachments m . The correlation amount $C_m(r)$ is the probability that the couples of points on the reconstructed attractor in the m -dimensional lags space are within the limits of distance r from each other. If the graph of the function $\ln C_m(r)$ relatively $\ln(r)$ has clearly defined linear area, this indicates to the existence of self-similar geometry of the attractor, which, in turn, points to the chaotic process. The correlation dimension D_2 of the attractor, which characterizes the dependence of the probability that two random points from attractor lie within the same cell that determines the dynamic heterogeneity of the attractor are finding as follows. We are considering the correlation integral $C(r)$, which shows the relative number of pairs of points of the attractor, which are at a distance of no greater than r :

$$C(r) = \frac{2}{m(m-1)} \sum_{i=0}^{m-2} \sum_{j=i+1}^{m-1} \theta(r - p(x_i, x_j)) \quad (7)$$

where, θ – the Heaviside function; p – the distance in n -dimensional phase space; m – the number of points x_i on the attractor. On the quite small scale lengths and when embedding dimension m not less than topological dimension of the attractor is performed dependence:

$$C(r) \approx r^{D_2} \quad (8)$$

where D_2 — the target correlation dimension of attractor. Having taking the logarithm of equation:

$$\ln C(r) \approx D_2 \ln r \quad (9)$$

The expression (9) gives the desired estimation of the attractor dimension as a tangent of slope angle of the straight line, that approximates the graph of correlation integral $C(r)$ in double logarithmic scale.

To calculate the correlation entropy we also calculated the correlation integral (7), but we consider not only its dependence on the distance r , but also on the dimension of the phase space n . In this case consider that

$$C(r, n) \approx r^{D_2} \exp(-nK_2) \quad (10)$$

whence

$$K_2(r, n) = \ln \frac{C(r, n)}{C(r, n+1)} \quad (11)$$

Entropy K_2 is approximated in an acceptable range of values r and n .

Materials and methods

The object of the research are time series of functioning of distillation unit. The researched material was the time series of distillation, rectification and epuration columns of distillation department of Krasnoslobodsk distillery. Detailed description of distillation unit are shown in [12].

The temperature of distiller's wort, the temperature at the control 16th plate of RC, the bottom and the top temperature of DC, the bottom temperature of EC, the temperature of the bottom of RC are measured by the resistance thermal converter of explosive performance TSP-1088. Conductors of resistance thermal converter to the control point are laid in metal pipes to prevent them from damage and the emergence of sparks.

For measuring pressure of the top and the bottom of distillation column, the top and the bottom pressure of epuration column (EC) and pressure of the bottom of RK is used DPP-2-13-001-0116 device. The output signal of this sensor is a standard pneumatic 20-100kPa.

The maximum error for all measurement channels is less than 1%.

The investigated materials. The fractal analysis of the functioning of distillation unit (DU) were conducted by processing of time series of the basic technological variables, such as the expenses of alcohol from rectification column (RC), the expenses and temperature of distiller's wort, the temperature at the control 16th plate of RC, the top and the bottom pressure of distillation column (DC), the bottom and the top temperature of DC, the bottom and the top pressure of epuration column (EC), the bottom temperature of EC, the temperature and the pressure of the bottom of RC.

For the processing and analysis of distillation unit are used data of 2 months of the distillery. The complete guidance of data in this article is considered inappropriate because of significant cluttering the space and for the improvement of the perception of information.

The procedure of conducting research.

We have converted the time series of technological variables to normalized form (Fig. 1) of varying duration and in different periods of DU operation, in order to establish variation of the results.

Then the time series of technological parameters processed using techniques of Hurst and the Fractan software package, which allows to conduct computation of the correlation functions, the correlation dimension, the Hurst exponent, the correlation entropy. Moreover to determine the fractal dimension and other characteristics that allow us to make conclusions about the self-similarity of the processes [13].

Processing of investigation. The correlation dimension used for the analysis of the behavior of chaotic component of the object. When the correlation dimension is not growing monotonically and has a maximum, so that indicates that the process is predictable and it defined by a certain number of parameters. This is the dimension of space of attachments m as the smallest intact dimension of the space that contains the full attractor, that is corresponds to the number of independent variables that uniquely identifies established movement of a dynamical system.

Correlation entropy K_2 allows to evaluate the average velocity of losing information about the state of the dynamic system over time and to determine the average time of predictability of system behavior, which also depends on the dimension of space attachments m . If $K_2=0$, the mode of oscillations of dynamic system is periodic, that corresponds to the regular process, and if $K_2>0$, then the system is in the mode of chaotic oscillations [14].

— Automatization of technological processes —

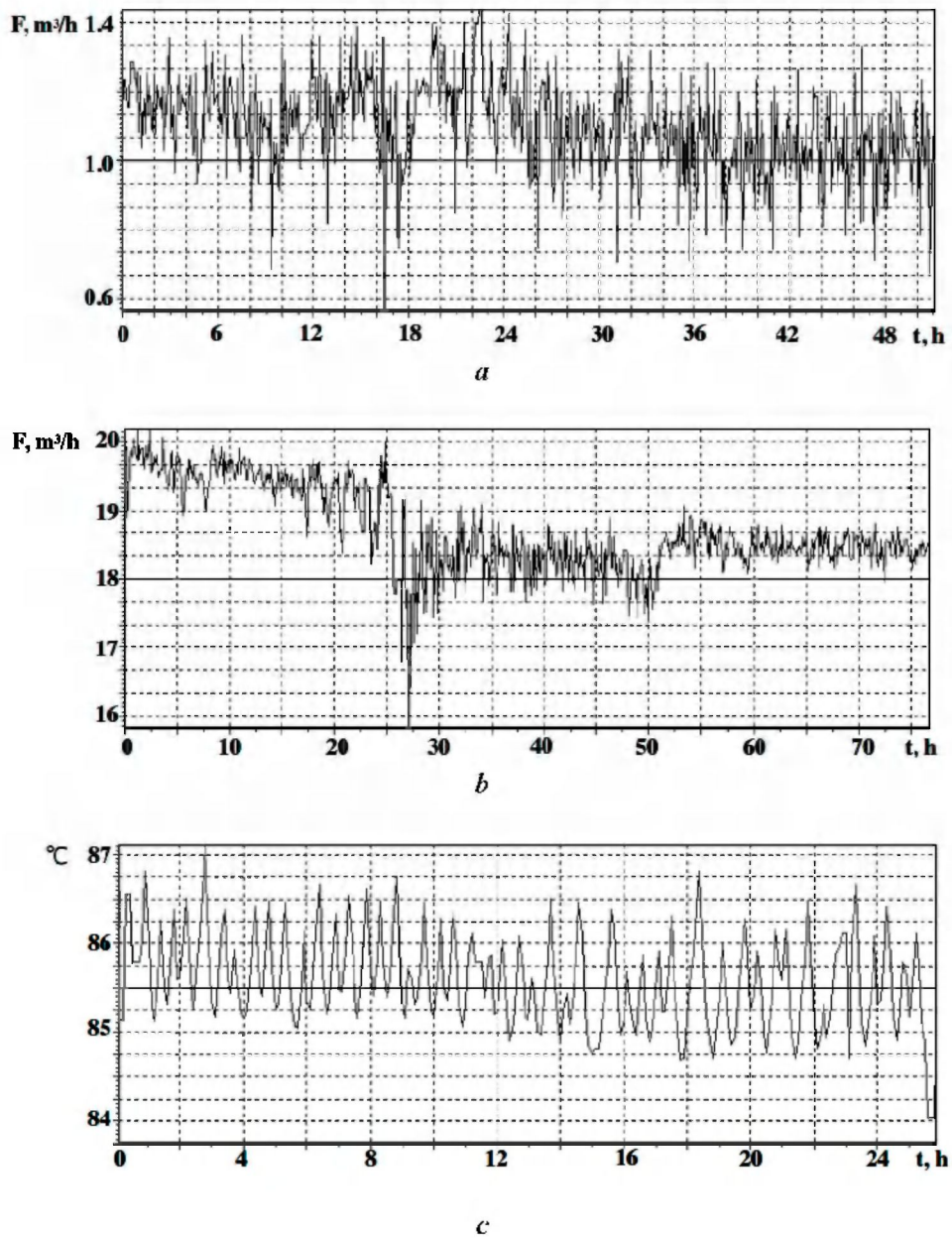


Fig.1. The normalized time-series of technological variables of DU of:
a - the alcohol expences from RC; b - the expenses of distiller's wort at DC; c - the temperature on the 16th plate of (RC)

Results and discussion

Determination of correlation dimension is shown in Fig. 2. As seen from the graphs, the curve have a pronounced maximum indicating that the process is predictable and dependent on the completely certain number of parameters, as indicated by dimension of the space attachments, which completely corresponds to the number of variables that drive the system in a particular direction (x-axis on the graphs of Fig. 2).

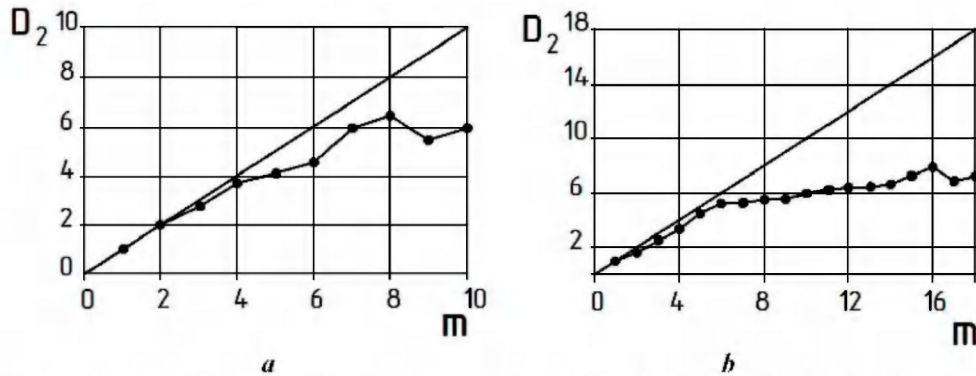


Fig. 2. The dependence of the correlation dimension D_2 from the dimension of the space attachments m for time series of:
a - the alcohol expenses from RC; *b* - the temperature on the 16 plate of (RC)

The dependence of the correlation entropy K_2 from the space dimension of attachment m is shown on Fig. 3

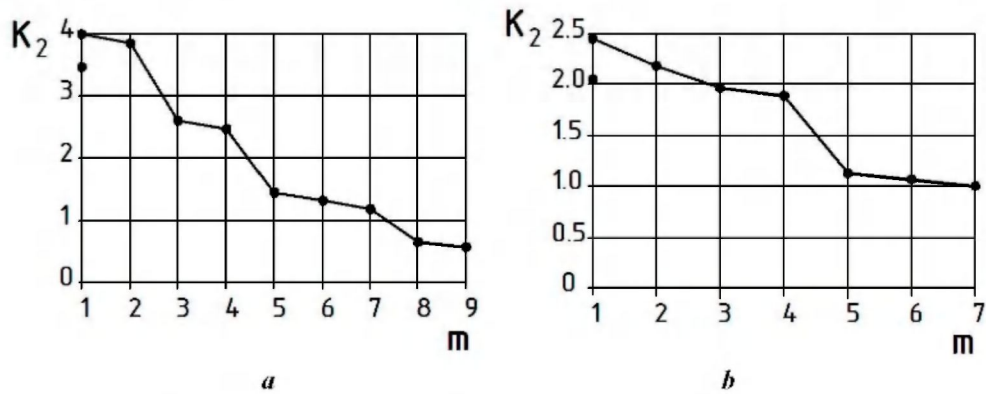


Fig.3. The dependence of the correlation entropy K_2 from the dimension of the space attachments m for time series:
a - the alcohol expenses from RC; *b* - the temperature on the 16 plate of (RC)

Particular importance of fractal time series analysis is that it takes into account not only the behavior of the system during the measurement, but also its history. Fractal dimension is an indicator of the curve complexity of time series.

We can learn how to predict system behavior by analyzing alternation of sections with different fractal dimension. And, what is most importantly, to diagnose and predict unstable states.

The essential aspect of this approach is the availability of critical value of fractal dimension of the temporal curve, upon approaching to which the system loses stability and goes into the unstable condition, parameters quickly grow or fall, depending on current trends.

Here is analyzed the dynamics of change of time series on the example of a bottom pressure of DC.

The fractal dimension of certain size can be used as an "indicator" of disaster. Analysis of experimental data shows that the trend line for the time series is well described by the equation:

$$\bar{y}(t) = \bar{y}(t_0) + \frac{K_f(t_0)(t-t_0)}{(D-D_0)^\beta}, \quad (12)$$

where $\bar{y}(t_0)$ - the average value of the quantity of the period that preceding to prediction period; K_f and β coefficients; t_0 - the period of time that precedes the prediction period; t - the time for which is doing the forecast; D_0 - the fractal dimension to the period that precedes the prediction period.

Moreover, the value of fractal dimension can indicate to the number of factors that affect the system [11]. If fractal dimension is less than 1.4, then one or more of the forces affects system, which drives the system in one direction. If the dimension is about 1.5, the forces acting on the system are multidirectional, but more or less compensate each other. If the fractal dimension is much more than 1.6, the system becomes unstable and ready to move to a new state.

Fig. 4 shows the dynamics of changes in fractal dimension of time series.

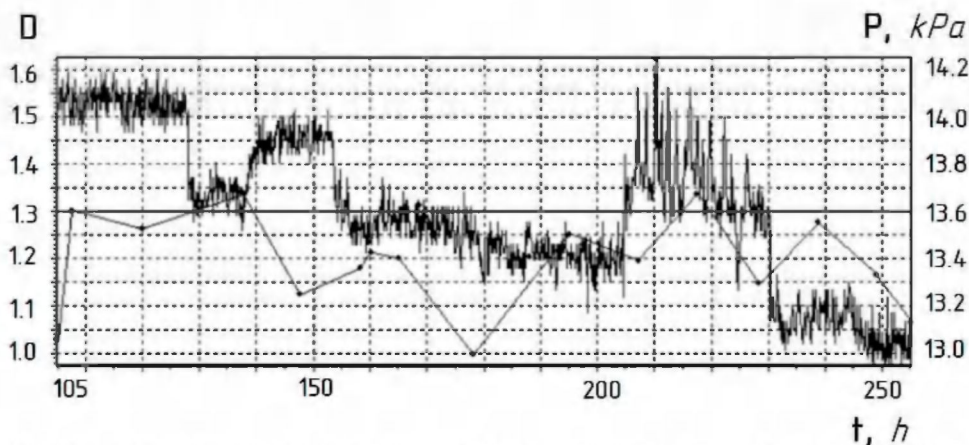


Fig.4. Dynamics of fractal dimension change of the time series of pressure in the bottom of MC

The research of the dynamics shows that during quite stable periods and slow ups the fractal dimension of the time series remains rather low, while in periods of abrupt changes the total fractal dimension is increased.

Another method of fractal analysis is based on the algorithm of *RS* – analysis of time series.

The Hurst exponent is a measure of persistency, that is, the tendency of process to trends [7]. The value of $H > \frac{1}{2}$ means that the dynamic of the process directed in a certain side in the past is likely to cause continuation of movement in the same direction. If $H < \frac{1}{2}$, it is predicted, that the process will change the direction. $H = \frac{1}{2}$ means uncertainty – Brownian motion [15].

To determine the fractal characteristics of chaotic information flows are calculated value of the Hurst exponent of time series for the main process variables of distillation department of distillery over a certain period (fig.5).

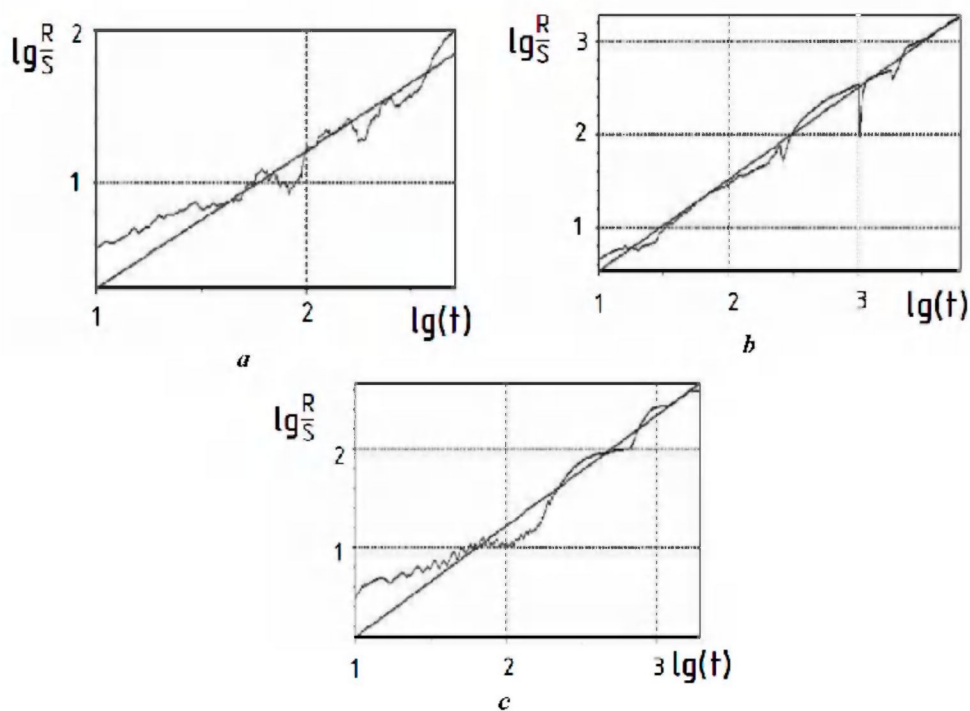


Fig.5. Calculation of the Hurst exponent of

- a - the alcohol expenses ($H=0.8992$);
- b - the pressure in the bottom of Distillation (Mash) Column (DC) ($H=0.9830$);
- c - the temperature on the 16th plate of rectification column (RC) ($H=0.9621$)

The generalized research results are presented in Table 1.

Table 1

The generalized research results

The observed parameter	H/D	Correlation dimension, D_2	The dimension of the phase space, n
Expense of distiller's wort, $F, m^3/h$	0,83/1,17	4.563	5
Temperature of distiller's wort, $^{\circ}C$	0,84/1,16	5.468	7
Pressure of the bottom of DC P, kPa	0,809/1,19	3.233	6
Pressure of the top of DC P, kPa	0.82/1,18	2.334	4
Temperature of the bottom of DC, $t, ^{\circ}C$	0,76/1,24	2.474	7
Temperature of the top of DC, $t, ^{\circ}C$	0,725/1,275	2.205	4
Pressure of the bottom of EC P, kPa	80,86/1,14	2.185	3
Temperature of the bottom of EC, $t, ^{\circ}C$	0,722/1,278	5.712	7
Pressure of the bottom of RC P, kPa	0,86/1,14	5.343	3
Temperature of the bottom of RC, $t, ^{\circ}C$	0,73/1,27	3.873	5
The temperature in the control plate of RC, $t, ^{\circ}C$	0,85/1,15	4.754	6

Conclusions

The computed exponents of Hurst testify about characteristic of the persistency of considered variables and the possible sufficiently deep their predictability.

After analyzing the Hurst exponent we can conclude that the estimated indicators of Hurst show that the time series data, and in turn distillation unit itself is a complex nonlinear dynamic control object, which has persistently characteristic of temporal behavior ($0.5 < H < 1$) [16,17].

The results of research pointed to the presence of intermittency in the technological processes that requires the monitoring systems creation for diagnostic regimes of operation of distillation unit. The operative identification of deterministic, stochastic or chaotic regimes by the methods of nonlinear dynamic makes it possible to implement resource-saving control strategies in the algorithms of control of distillation technological processes, which will increase the efficiency of distillation unit functioning.

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