

**THE OPTIMAL CONTROL MODELS OF INTERURBAN BUS TRANSPORT**

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**Abstract.** *The models for calculating the number of buses of different types depending on the variable passenger time and a specified interval of vehicles, minimizing the expenses of motor company which is considered to be optimal for the carrier and the passenger range of buses on the route. Analyzing the influence of the speed of vehicles on the total costs of MTE. The proposed model can be used to make effective management decisions regarding the organization of the rolling stock for enterprises passenger transport.*

**Keywords:** mathematical programming, optimization, cost, motor company, buses, intercity transportation, optimal plan.

**I. INTRODUCTION. STATEMENT OF THE PROBLEM**

For motor transport enterprises (MTE) which are engaged in passenger transportation, the main tasks are to provide quality competitive services and minimize costs for passenger services. Due to the recent rising prices for fuel and lubricants (FL), there is a need in the economy and an efficient use of resources MTE. The objective is to develop mathematical model optimization of rolling stock of different types depending on traffic flow changes in time to minimize maintenance costs MTE vehicles.

**II. THE BASIC PRINCIPLES OF OPTIMIZATION OF MTE**

The theory of the organization of work of the rolling stock with the use of mathematical modeling has been considered in [1-4]. Exploration companies optimize passenger transport and management of rolling stock regarding fluctuations in passenger traffic studied in [5-7]. To optimize MTE used multicriteria (vector) methods (e.g., principal criterion for Pareto optimum, methods of successive concessions). Optimization of MTE may be aimed at maximizing profits, the volume of passengers, profitability, productivity, and minimizing the cost, payback period and expenditure of resources. MTE competitiveness depends on its level of profitability and quality of passenger service. Optimization of MTE is, on the one hand, to meet the demand of consumers of transport services, in reducing the waiting time in the queue (ride comfort for passengers), and the other - in reducing the cost of passenger traffic on the route. In previous studies, the authors found the optimal range of movement of vehicles (MV) on the route for the  $i$ -th hour of MTE for one type of bus capacity ( $N$ ):

$$\tau_i = \sqrt[3]{\frac{2NS^2}{F_i^2 H}} \quad (1)$$

where  $N$  – is the capacity of the vehicle,  $S$  – the net cost of transport 1 passenger on the route for one hour  $F_i$  – the average passenger flow for the  $i$ -th hour [8],  $H$  – average cost of labor for one hour the inhabitants of the region. Model (1) allows you to get the best value traffic  $\tau$ , movement of vehicles (MV) with a mean of passenger traffic  $F_c$  on the route.

**III. ROLLING STOCK MANAGEMENT MODEL MTE**

To satisfy the passenger at the same time to avoid loss of the enterprise, the carrier must make rational use of the rolling stock of different sizes depending on the amplitude-frequency characteristics of passenger transport demand. Consider the problem of a rational choice of four types of rolling stock for 1 hour of work for a private company to transport passengers "Vladis" (city Chernihiv), which serves passengers on the routes "Kyiv-Chernihiv", "Kyiv-Chernihiv".

It is known that the costs of any enterprise are divided into constant that cannot "undo" (e.g., land rent, taxes, etc.) and variables related to the production of goods and services. In general, the profit MTE " $P$ " can be expressed as the difference between revenues and costs of two types:

$$P = D - V_1 - V_2, \quad (2)$$

where  $D$  is the value of the income associated with the volume of passenger traffic and the cost of tickets,  $V_1$  – vehicle maintenance costs (including salary drivers) that depend on the choice of the structure of the rolling stock,  $V_2$  – production costs (administration, production facilities, social contributions, etc.), which do not depend on the types of

buses running on route. Then the task of maximizing profits ( $P \rightarrow \max$ ) in the model (2) is reduced to the problem of minimizing expenses ( $V_1$ ) for maintenance of the rolling stock, which can be represented as:

$$V_1 = \sum_{j=1}^4 \left( \left( \frac{s_0}{\tau_0} \right) \cdot \gamma_j + z \right) \cdot K_{1j} \rightarrow \min \quad (3)$$

and restrictions on the number  $F_1$  of transported passengers per hour and the total number of seats in buses:

$$F_1 \leq \sum_{j=1}^4 (K_{1j} \cdot N_j),$$

as well as the average time interval  $\bar{\tau}_1$  of the MV, which shall not be greater than the specified  $\tau_1^+$ :

$$\bar{\tau}_1 = \frac{1}{K_1} \leq \tau_1^+,$$

where  $\gamma_j$  – is the direct material costs (UAH/km),  $z$  – is the drivers salary (UAH/hour)  $K_{1j}$  and  $N_j$  ( $j = 1, 2, 3, 4$ ) – accordingly, the number of MV and the number of vehicle locations therein for 1 hour,  $K_1 = \sum_{j=1}^4 (K_{1j})$  – the total MV,  $\tau_0$  and  $s_0$  – respectively MV (hours) and the length of the route in miles.

Entering symbols  $a_j = \frac{s_0}{\tau_0} + z$ ,  $x_j = K_{1j}$

,  $c_j = N_j$  the task can formulate as a linear programming problem with the target function:

$$V_1 = a_1 x_1 + a_2 x_2 + a_3 x_3 + a_4 x_4 \rightarrow \min, \quad (4)$$

the limitations:

$$\begin{cases} c_1 x_1 + c_2 x_2 + c_3 x_3 + c_4 x_4 \geq F_1, \\ x_1 + x_2 + x_3 + x_4 \geq \frac{1}{\tau_1^+}, \\ 0 \leq x_j \leq \mu_j, j = 1, 2, 3, 4, \end{cases} \quad (5)$$

where  $\mu_j$  – restriction on the availability of the relevant type of vehicle.

The average fill factor of MV  $\bar{\varepsilon}_1$  defines ratios:

$$F_1 = \sum_{j=1}^4 (\varepsilon_{1j} \cdot K_{1j} \cdot N_j),$$

$$\bar{\varepsilon}_1 = \frac{F_1}{\sum_{j=1}^4 (K_{1j} \cdot N_j)} \quad (0 < \bar{\varepsilon}_1 \leq 1)$$

where  $\varepsilon_{1j}$  – fillfactor for  $j$ -th MV.

The simulation results have shown an adequate assessment to support decision-making, which is confirmed by the actual performance of the rolling stock of the MTE in Chernigov. According to the study, the authors of the proposed model, when using an optimization cost MTE decreases ranging from 2 to 15%, depending on the interval of MV. The results of modeling the structure of the rolling stock in comparison to the actual choice of the carrier have shown that when using the proposed models, for example, for an interval of 20 minutes, costs reduced by 12.5%, for 15 minutes, at 12% for 5 minutes at 9.7%.

Using the features "Solver" in MS Excel, we obtain the optimal production plan on the route for the four types of buses for a given passenger flow  $F_1$  and set interval movement of the vehicle  $\tau_1^+$ . Model (6-7) allows the carrier to optimize efficient use of buses of different capacity and at the same time take into consideration the interests of passengers due to the restriction on the optimal time interval (1).

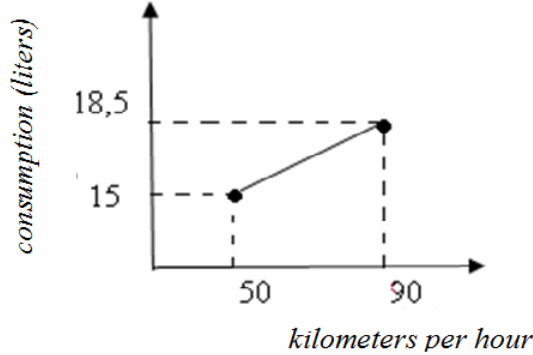
#### V. ACCOUNT OF TECHNOLOGICAL FEATURES OF ROLLING STOCK IN THE OPTIMAL MANAGEMENT OF MTE

The different types of rolling stock have different economic and technical characteristics. For example, table 1 shows the cost of fuel at the rate of four types of rolling stock from 60 km/h to 90 km/h.

**Table 1.** Fuel costs depending on the speed of the bus four types

Types of bus	speed, km/h							
	"Etalon" (30 seats)		"Bogdan" (19 seats)		"Mercedes" (18 seats)		"Volks-wagen" (19 seats)	
	60	90	60	90	60	90	60	90
The cost of fuel, l	15	18,5	15	16,5	14	11,5	13	11

Due to the rising cost of petrol, oil and lubricants need to find not only the best types of vehicle on the route, but the speed of their movement. The dependence of fuel consumption from the speed of MV could be close to linear (fig. 1).



**Fig.1** Linear approximation of fuel consumption on the speed of the bus "Etalon" (30 seats) fuel

We get a system of equations with two unknowns for each type of bus stop:

$$\begin{cases} \alpha_1 + \beta_1 \cdot 60 = 15, \\ \alpha_1 + \beta_1 \cdot 90 = 18,5, \end{cases} \Rightarrow \alpha_1 = 8, \beta_1 = 0,117,$$

$$\begin{cases} \alpha_2 + \beta_2 \cdot 60 = 15, \\ \alpha_2 + \beta_2 \cdot 90 = 16,5, \end{cases} \Rightarrow \alpha_2 = 12, \beta_2 = 0,05,$$

$$\begin{cases} \alpha_3 + \beta_3 \cdot 60 = 14, \\ \alpha_3 + \beta_3 \cdot 90 = 11,5, \end{cases} \Rightarrow \alpha_3 = 19, \beta_3 = -0,083,$$

$$\begin{cases} \alpha_4 + \beta_4 \cdot 60 = 13, \\ \alpha_4 + \beta_4 \cdot 90 = 11, \end{cases} \Rightarrow \alpha_4 = 17, \beta_4 = -0,67.$$

Then the cost of fuel at 1 km  $r_1, r_2, r_3, r_4$  depending on the speed of movement for each of the four types of buses "Bogdan", "Etalon", "Mercedes", "Volkswagen" will be:

$$\begin{aligned} r_1 &= 0,08 + 0,001 \cdot v_1, \\ r_2 &= 0,12 + 0,0005 \cdot v_2, \\ r_3 &= 0,19 - 0,001 \cdot v_3, \\ r_4 &= 0,17 - 0,001 \cdot v_4. \end{aligned}$$

Taking into account other direct material expenses on maintenance of the rolling stock of each of the four types of buses on 1 km (replacement of oils, tires and the like), we obtain the following dependencies:

$$\begin{aligned} \gamma_1 &= 1,3 + 0,0175 \cdot v_1, \\ \gamma_2 &= 1,9 + 0,0075 \cdot v_2, \\ \gamma_3 &= 2,95 - 0,0125 \cdot v_3, \\ \gamma_4 &= 2,65 - 0,01 \cdot v_4, \end{aligned}$$

Where  $\gamma_1, \gamma_2, \gamma_3, \gamma_4$  – are the total cost of ownership of the Etalon buses "Bogdan", "Mercedes", "Volkswagen" for 1 km.

Let us denote the desired speed for the four types of buses through  $x_5, x_7$ . Then the model to minimize the cost of maintenance of the rolling stock (3) can be thought of as a problem of nonlinear programming in the form of:

$$\begin{aligned} V_1 &= s_0 \left( (1,3 + 0,0175 \cdot x_5 + \frac{z}{x_5}) \cdot x_1 + \right. \\ &+ (1,9 + 0,0075 \cdot x_6 + \frac{z}{x_6}) \cdot x_2 + \\ &+ (2,95 - 0,0125 \cdot x_7 + \frac{z}{x_7}) \cdot x_3 + \\ &\left. + (2,65 - 0,01 \cdot x_8 + \frac{z}{x_8}) \cdot x_4 \right) \rightarrow \min, \end{aligned} \quad (6)$$

the limitations:

$$\begin{cases} c_1 x_1 + c_2 x_2 + c_3 x_3 + c_4 x_4 \geq F_1, \\ x_1 + x_2 + x_3 + x_4 \geq \frac{1}{\tau_1^+}, \\ 0 \leq x_j \leq \mu_j, j = 1, 2, 3, 4, \\ 60 \leq x_j \leq 90, j = 5, 6, 7, 8, \end{cases} \quad (7)$$

Where  $\mu_1 = 18, \mu_2 = 3, \mu_3 = 23, \mu_4 = 6$ .

In the model (6)  $s_0 \cdot \frac{z}{x_5}$  displays the driver's

salary from time to time, for any driver performs the carriage of passengers. For MTE "Vladis" km, salary of driver for decorated flight is 100 UAH. And therefore does not depend on the speed of the bus on the route. Then the cost of MTE "Vladis" for servicing rolling stock would be:

$$\begin{aligned} V_1 &= (150 \cdot (1,3 + 0,0175 \cdot x_5) + 100) \cdot x_1 + \\ &+ (150 \cdot (1,9 + 0,0075 \cdot x_6) + 100) \cdot x_2 + \\ &+ (150 \cdot (2,95 - 0,0125 \cdot x_7) + 100) \cdot x_3 + \\ &+ (150 \cdot (2,65 - 0,01 \cdot x_8) + 100) \cdot x_4 \rightarrow \min \end{aligned} \quad (8)$$

at the constraints (7).

Table 2, 3 shows the optimal decoupling linear programming problem (4), (5), which corresponds to Plan 2 and nonlinear programming (6), (7), corresponds to plan 3 under different restrictions on passenger traffic, the interval of motion, the speed of their vehicle movement.

**Table 2.** Optimal structure of rolling stock plans for the route "Chernihiv-Kyiv"

Number of transported passengers per hour and optimal range of movement of vehicles	Condi- tion 1 $F1=$ 282 pass./ hour $\tau_1^+ =$ 0,08 hour			Condi- tion 2 $F1=$ 198 pass./ hour $\tau_1^+ =$ 0,16 hour			Condi- tion 3 $F1=$ 102 pass./ hour $\tau_1^+ =$ 0,25 hour			Condi- tion 4 $F1=$ 66 pass./ hour $\tau_1^+ =$ 0,5 hour		
	Plan 1			Plan 1			Plan 1			Plan 1		
	Plan 2			Plan 2			Plan 2			Plan 2		
	Plan 3			Plan 3			Plan 3			Plan 3		
Optimal structure of the necessary rolling stock	Plan 1	Plan 2	Plan 3	Plan 1	Plan 2	Plan 3	Plan 1	Plan 2	Plan 3	Plan 1	Plan 2	Plan 3
"Etalon" (30 seats)	4	5	6	6	6	1	1	3	1	1	1	2
"Bogdan" (19 seats)	3	0	1	1	0	0	2	0	0	0	0	0
"Mercedes" (18 seats)	6	1	0	0	0	0	1	0	0	1	0	0
"Volkswagen" (19 seats)	1	6	5	0	1	6	1	1	3	1	2	1
$V_1$ UAH / hour	3235	2765,63	2409,79	1820	1797,5	1605,42	1136,88	1000,63	889,17	684,38	673,13	584,58

**Table 3.** The optimal speed for "Etalon" buses "Bogdan", "Mercedes", "Volkswagen"

Speed, km/h	$v_1$	$v_2$	$v_3$	$v_4$
Condition 1	60	-	90	90
Condition 2	60	-	-	90
Condition 3	60	-	-	90
Condition 4	60	-	-	90

In Table 4 percent decrease is designed for linear (4), (5) and non-linear (6), (7) model. Plan 4 corresponds to the model (8). The table shows that the optimum speed reduces cost of MTE for servicing

rolling stock approximately 10-15%. It is essentially a constant growth in fuel prices.

**Table 4.** Comparison of the effectiveness of the proposed models for the carrier's actual costs in percentage terms.

Plans	Plan 2	Plan 3	Plan 4
Conditions			
Condition 1	14,51	25,51	31,45
Condition 2	1,24	11,79	15
Condition 3	11,98	21,79	24
Condition 4	1,64	14,58	14

As you can see, using the proposed model (3) MTE decrease costs in the range of 1.2 - 14.5%, and the use of the model (6), which complements the model (3) restrictions on the rational speed - in the range of 11.8 - 25.5%. Consequently, the model (6) allows the carrier by almost a quarter to reduce maintenance costs of rolling stock in MTE constantly rising prices for fuel and lubricants. According to the authors, the use of model (8) is optimal for the problem of optimal control intercity passenger services.

## V. CONCLUSIONS

Mathematical model determine the optimal structure of the necessary rolling stock for ensure-demand for passenger transport, taking into account the conditions to minimize the cost of MTE. The model takes into account the economic interests of the carrier and the passenger that is expressed by the optimal interval movement of the vehicle, which was found by the authors in previous studies. The purpose of the follow-up studies of authors is to create software for the introduction of developed models in automated system management decision-making with respect to the rational choice of rolling stock MTE with statistical epicycles of passengers on long-distance routes in the region. Alsotasks, connected with creating it new lycoined MTE to transport passengers with optimum structure of the rolling stock in limiting the starting capital of the entrepreneur, as well as the challenge of planning for optimal acquisition of new vehicles by entering related financial restrictions, given their payback.

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**М. Г.Медведєв, Л. М.Олещенко. Моделі оптимального управління міжміськими автобусними пасажирськими перевезеннями.** Описані моделі для розрахунку кількості автобусів різних типів залежно від змінного пасажиропотоку в часі і заданого інтервалу руху транспортних засобів. У моделі мінімізації витрат автотранспортного підприємства враховується оптимальний для перевізника і пасажирів інтервал руху автобусів на маршруті. Проаналізовано вплив швидкості руху транспортних засобів на загальні витрати АТП. Запропоновані моделі можуть використовуватися для ухвалення ефективних управлінських рішень щодо організації роботи рухомого складу для підприємств пасажирського транспорту.

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**Н. Г. Медведєв, Л. М. Олещенко. Модели оптимального управления междугородными автобусными пассажирскими перевозками.** Описаны модели для расчета количества автобусов разных типов в зависимости от переменного пассажиропотока во времени и заданного интервала движения транспортных средств. В модели минимизации расходов автотранспортного предприятия учитывается оптимальный для перевозчика и пассажира интервал движения автобусов на маршруте. Проанализировано влияния скорости движения транспортных средств на общие издержки АТП. Предложенные модели могут использоваться для принятия эффективных управленческих решений относительно организации работы подвижного состава для предприятий пассажирского транспорта.

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