

Synthesis of modal control of a linear system for a given *LMI*-domain

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The classical approach to the synthesis of linear feedback (regulators) in the state space is associated with the canonical representation of the controlled object and the construction of a modal control (controller) that provides the given eigenvalues (mods) of the matrix of a closed system. Then the construction of the modal control reduces to the finding of the characteristic polynomial of the matrix A , the choice of the canonical basis, and the solution of the system of linear equations. But the construction of modal control can also be based on the application of the theory of linear matrix inequalities (*LMI*).

Let the control object be described by the equation

$$\frac{dx(t)}{dt} = Ax(t) + Bu(t), \quad (1)$$

where $x(t) \in R^n$ – state of the object, $u(t) \in R^m$ – control.

The synthesis task consists in choosing the law of control $u(t)$ from the class of linear feedback relations on the state of the form

$$u(t) = Kx(t), \quad (2)$$

where K – the matrix of the parameters of the regulator corresponding to the order in which the matrix of the closed system (1), (2) will be D -stable and the values of its roots lie in the given *LMI*-domain.

The problem of D -stability is reduced to finding matrices $X = X^T > 0$ and K , which satisfy the nonlinear inequality $M(A + BK, X) < 0$. When labeled $Z = KX$, the last inequality can be represented as a linear matrix inequality of the form

$$\begin{aligned} M(A + BK, X) &= P \otimes X + G \otimes ((A + BK)X) + G^T \otimes (X(A + BK)^T) = \\ &= P \otimes X + G \otimes (AX + BZ) + G^T \otimes (AX + BZ)^T = P \otimes X + G \otimes (AX) + G \otimes (BZ) + \\ &+ G^T \otimes (AX)^T + G^T \otimes (BZ)^T = M(A, X) + G \otimes (BZ) + G^T \otimes (BZ)^T < 0 \end{aligned}$$

with respect to unknown matrices K and Z . After these matrices are found, the desired matrix of the parameters of the regulator is as $K = ZX^{-1}$.

Literature

1. Лобок О.П. Застосування лінійних матричних нерівностей при синтезі модального керування багатомірними лінійними системами / О.П. Лобок, Б.М. Гончаренко, М.А. Сич // Журнал «Наукові праці НУХТ». Том 24, № 3. – К: НУХТ. 2018, с.16 – 25.