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THE MEAN-VALUE THEOREM FOR HOLOMORPHIC FUNCTIONS.

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There are many articles devoted to local mean value theorems for vector-valued functions, in particular holomorphic functions. This report relates directly to this area and will be studied the problem of representing the remainder of the Taylor expansion for a holomorphic function in Lagrange form. We find out when and how the expansion well-known in the case of real-valued functions on an interval of the real axis can be transferred to holomorphic functions in a complex domain. Our theorems not only cover the some known results but also imply the following intuitively clear fact: If f is a holomorphic function in a neighborhood of the real axis and f takes real values at real values of the argument then the mean value in the remainder of the Taylor expansion, written down in Lagrange form, can be localized more precisely than without using the holomorphy of f . Henceforth f is a holomorphic function in a domain D of the complex plane $\mathbb{C}$, is the boundary of $\overline{D}$, and is the closure of D. We denote by $U(\alpha;\, r) := \{z \in \mathbb{C} : |z - \alpha| < r\}$ the open disk of radius $r > 0$ centered at α and let $\arg z$ — stand for the argument of a nonzero complex number z and $-\pi < \arg z \leq \pi$.We suppose that the points z_0 and z_1 belong to D. In this report we study the following question: When is the remainder $Q_n(z_0; z_1; f)$ of the Taylor expansion

$$f(z_1) = \sum_{k=0}^{n-1} \frac{(z_1 - z_0)^k}{k!} f^{(k)}(z_0) + Q_n(z_0; z_1; f),$$

representable in Lagrange form $Q_n(z_0; z_1; f) = \frac{(z_1-z_0)^n}{n!} f^{(n)}(\xi)$ and where does ξ lie?

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