THE SOLUTION OF THE PROBLEM OF CONTACT OF A PUNCH WITH MULTIPLY CONNECTED HALF-PLANE

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1. Introduction

Analytical solutions for all types of problems about denting of rigid punches into uniform elastic semi-infinite domain are practically received [1-3]. However mathematical modeling of system brings to the schemes that considerably complicate solution of the problem. Among of the mass or rigid inhomogeneity of a solid body, complex boundary conditions etc. There are a lot of numerical procedures are usually used for solution of the aforesaid class of problems. The most drawback of these procedures is that they are not capable to solve a problem for semi-infinite domains without simplification of mathematical models of the process considered [4-5].

2. Statement of the problem

Consider an elastic problem about denting the punch with lubricant without a friction into semi-infinite domain. Boundary conditions are the following: the displacements at the restricted section of domain are prescribed. The remaining part of the domain is free. Its' boundary is fixed. (fig. 1).

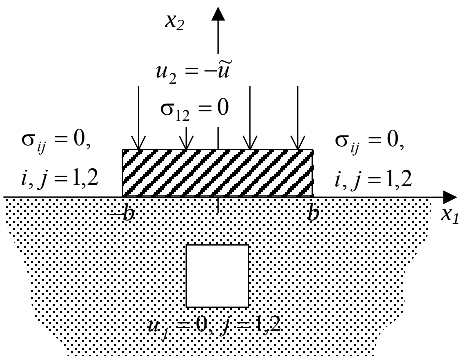

Fig. 1 The problem about denting of the punch into multiply connected semi-infinite domains

Governing equations of the problem is following [6]:

$$G\Delta u_j + \frac{G}{1-2\nu}\frac{\partial}{\partial x_j}\left(\frac{\partial u_1}{\partial x_1}+\frac{\partial u_2}{\partial x_2}\right)=0 \qquad (1)$$

where j = 1,2, ν – is the Poisson's ratio, G – is the shift modulus of elasticity.

3. Method of the solution

For solution of the problem above we use an analytically-numerical method BEM. The integrated equation is the following [7]:

$$\begin{aligned} & c_{ij}(\xi)u_j(\xi)+\int_\Gamma p*_{ij}(\xi,x)u_j(x)d\Gamma(x)= \\ & =\int_\Gamma u*_{ij}(\xi,x)p_j(x)d\Gamma(x)+\int_\Omega u_{ij}(\xi,x)b_j(x)d\Omega(x),\ \xi\in\Gamma \end{aligned}, \qquad (2)$$

where $c_{ij}(\xi)=\frac{\delta_{ij}}{2}$ – are the functions for smooth surfaces, δ_{ij} – is the Kroneher's symbol, $u*_{ij}(\xi,x)$ – is the Calvin's fundamental solution:

$$u^*_{ij}(\xi,x)=\frac{-1}{8\pi(1-\nu)G}\left[(3-4\nu)ln(r)\delta_{ij}-r,_i\,r,_j\right],$$

$p*_{ij}(\xi,x)$ – is the fundamental stress:

$$p^*_{ij}(\xi,x)=\frac{-\left[(1-2\nu)\delta_{ij}+2r,_i\,r,_j\right]\frac{\partial r}{\partial\overline{n}}+(1-2\nu)\left(r,_i\,n_j-r,_j\,n_i\right)}{4\pi(1-\nu)r},$$

$r=r(\xi,x)$ – is a distance between a loaded point ξ and certain point x of the plane.

We may determine displacements stresses inside of internal point of domain by using Somigliana's formulas with the help of numerical integration [7].

For solution of this class of problems the following scheme are usually used: boundary conditions at the infinity are transferred to the restricted contour. This procedure gives the problem with restricted boundary and BEM could be useful for solution of the problem. Here we propose new technique that allows to overcome aforesaid artificial simplification [8]. Consider the problem in the form:

$$c_{ij}(\xi)\Theta(\xi,\Gamma-\Gamma_5)u_j(\xi)+\int_{\Gamma}\hat{p}*_{ij}(\xi,x)u_j(x)d\Gamma(x)= \\ =\int_{\Gamma}\hat{u}*_{ij}(\xi,x)p_j(x)d\Gamma(x)+\hat{b}_i(\xi), \qquad (3)$$

where

$$\hat{p}*_{ij}(\xi,x)=\Theta(\xi,\Gamma-\Gamma_5)p*_{ij}(\xi,x),$$

$$\hat{u}*_{ij}(\xi,x)=\left(u*_{ij}(\xi,x)-\delta_{j2}\Theta(\xi,\Gamma_1\cup\Gamma_2)\left(\int_{\Gamma_5}p*_{ij}(\xi,y)F_j(y,x)d\Gamma(y)+\right.\right. \\ \left.\left.+c_{ij}(\xi)\Theta(\xi,\Gamma_5)F_j(\xi,x)\right)\right),$$

$$\hat{b}_i(\xi)=-\int_{\Gamma_3}\left(\int_{\Gamma_5}p*_{ij}(\xi,y)F_j(y,x)d\Gamma(y)+c_{ij}(\xi)\Theta(\xi,\Gamma_5)F_j(\xi,x)\right)p_2(x)d\Gamma(x),$$

$$F_1(\xi,x)=\frac{1}{2\pi G}\left\{(1-2\nu)\left(\operatorname{arctg}\frac{\xi_2}{\xi_1-x_1}+\frac{\pi}{2}\right)+\frac{\xi_1\xi_2}{(\xi_1-x_1)^2+\xi_2^2}\right\},$$

$$F_2(\xi,x)=\frac{1}{2\pi G}\left\{-2(1-\nu)\left[ln\sqrt{(\xi_1-x_1)^2+\xi_2^2}-ln|L-x_1|\right]+\frac{\xi_2^2}{(\xi_1-x_1)^2+\xi_2^2}\right\},$$

$$\Theta(\zeta,S)=\begin{cases}1, \text{ if } \zeta\in S,\\ 0, \text{ if } \zeta\notin S,\end{cases},$$

Γ – is the boundary of domain; Γ_5 – is the boundary of artificial internal contour.

$\hat{b}_i(\xi)$ in the formula (3) represents an “additional” loading at the semi-infinite domain.

The discrete form of equation (3) is constructed for each of boundary elements. Thus, the boundary Γ is really approximated with the help of the constant boundary elements. To achieve an acceptable accuracy of the solution the number of boundary elements of Γ has been determined experimentally.

Using the boundary conditions we may obtain al system of the linear algebraic equations for the unknowns that should be determined. Gauss method is applied for calculation of integrals. At the internal domain, solution of the problem could be now determined by the formula (3). At the infinite domain such the solution is obtained by the analytical Flaman’s solution [6].

4. Application of the method to the problem with a square hole

Here we present the results of the solution above and compare them with analytical results obtained in [9].

Numerical values of stress under the punch are presented at fig. 2 and in tab. 1. The coordinates of hole are following $(0;-0{,}5b)$. Linear dimensions of a hole varies from $0{,}5b$ to $0{,}001b$.

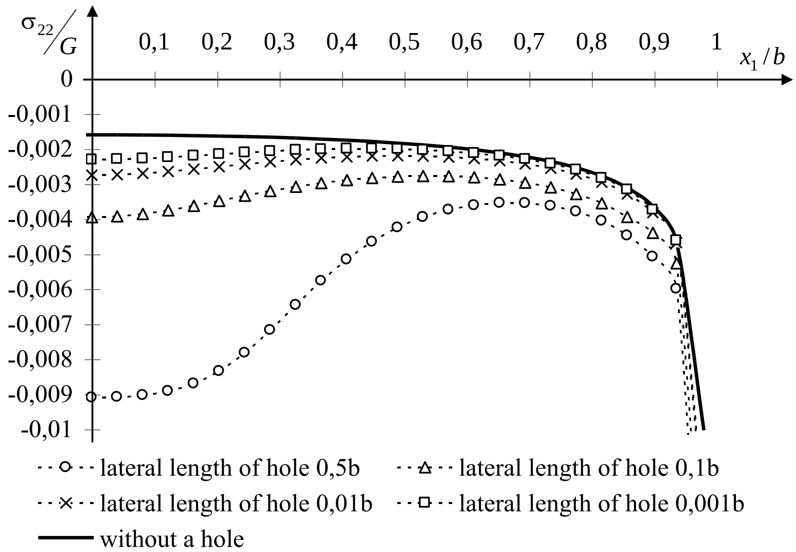

Fig. 2 Normal stress under the punch

Table 1

Values of pressure(stress) under the punch

x_1 / b	$\sigma_{22} / G \cdot 10^{-3}$				
	Lateral length of hole				Without a hole
	0,001*b*	0,01*b*	0,1*b*	0,5*b*	
0	-2,31027	-2,74990	-3,95035	-9,10285	-1,60299
0,04081	-2,30231	-2,73776	-3,92694	-9,08789	-1,60433
0,08163	-2,27948	-2,70286	-3,85953	-9,03501	-1,60836
0,12244	-2,24472	-2,64938	-3,75594	-8,91993	-1,61514
0,16326	-2,20218	-2,58322	-3,62724	-8,70411	-1,62479
0,20408	-2,15643	-2,51090	-3,48548	-8,34659	-1,63745
0,24489	-2,11175	-2,43847	-3,34187	-7,82899	-1,65334
0,28571	-2,07174	-2,37097	-3,20551	-7,17909	-1,67272
0,32653	-2,03908	-2,31213	-3,08302	-6,46734	-1,69595
0,36734	-2,01569	-2,26451	-2,97866	-5,77462	-1,72349
0,40816	-2,00286	-2,22972	-2,89488	-5,16018	-1,75591
0,44897	-2,00150	-2,20877	-2,83289	-4,65156	-1,79397
0,48979	-2,01241	-2,20240	-2,79328	-4,25228	-1,83863
0,53061	-2,03648	-2,21136	-2,77648	-3,95372	-1,89118
0,57142	-2,07495	-2,23675	-2,78326	-3,74384	-1,95331
0,61224	-2,12971	-2,28027	-2,81512	-3,61184	-2,02739
0,65306	-2,20363	-2,34467	-2,87481	-3,55032	-2,11670
0,69387	-2,30118	-2,43433	-2,96711	-3,55639	-2,22609
0,73469	-2,42949	-2,55641	-3,10011	-3,63289	-2,36290

0,7755 1	-2,60038	-2,72285	-3,28763	-3,79085	-2,53905
0,8163 2	-2,83505	-2,95525	-3,55503	-4,05573	-2,77531
0,8571 4	-3,17075	-3,29167	-3,94715	-4,47634	-3,11213
0,8979 5	-3,74277	-3,82	-4,4	-5,08	-3,64251
0,936	-4,62	-4,68	-5,26519	-6	-4,65270
0,9795 9	-12,32913	-12,66011	-15,09543	-17,18857	-10,03

3. Conclusions

1. From the received results follows, that influence of the concentrator of stress as a hole with fixed boundary on the distribution of contact stresses under the punch is limited by size of the hole.

2. Reduction of the linear size of a hole results in reduction of stress under a punch, but it cannot isolate influence of the concentrator.

3. Presented algorithm can be used for the solution of problems for semi-infinite domains with multiconnectivity of any kind and when there are some domains are not crossed.

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Summary

This paper is devoted to solving of problems about smooth contact of a punch with multiply connected half-plane. Half-plane contains a square hole different sizes. The solution is obtained with help of a new analytically-numerical approach, which based on synthesis of a method of boundary elements and analytical solutions for half-plane. Researches of the contact stresses under a punch are presented.

Key words: integral presentations, Method of Boundary Elements, bounding contour, multiply connected body, punch, contact stresses, strainly-deformed state.