Theorem on the estimation of the est approximations for the generalized derivative in Banach spaces

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We consider a Banach space B with a complete minimal system $\{\varphi_m\}_m^\infty$ and let $\{\varphi_m^*\}_m^\infty$ be its conjugate system belonging to B^* and $\{\lambda_m\}_m^\infty$ is a sequence of complex numbers. Let us introduce the following notion

Definition. *If for the element $f \in B$ the sum of the series $\sum_{m=1}^\infty \lambda_m(f, \varphi_m^*)\varphi_m$ is some element $g \in B$, then the vector g is called the derivative of the vector f and is denoted by $\partial_\varphi^\lambda f$,namely*

$$\partial_\varphi^\lambda f = \sum_{m=1}^{\infty} \lambda_m(f, \varphi_m^*)\varphi_m \qquad (1)$$

. The subset of all vectors $f \in B$ having $\partial_\varphi^\lambda f$ - derivatives will be denoted by $V(\partial_\varphi^\lambda)$. Vector

$$T_n(\varphi_n) = \sum_{m=1}^{n} c_m\varphi_m \qquad (2),$$

where c_m are arbitrary complex numbers, we will call a polynomial of degree n according to the system $\{\varphi_m\}_m^\infty$. Note that due to the system $\{\varphi_m\}_m^\infty$ is minimal therefor the coefficients c_m in (2) are uniquely determined by the vector $T_n(\varphi_n)$ and $c_l = (T_n(\varphi_n), \varphi_m^*)$.

Let $E_n(f, \varphi_m) = \inf_{T_n(\varphi)} \|f - T_n(\varphi_n)\|)$ be the best approximations of the vector f by polynomials of degree n over the system $\{\varphi_m\}_m^\infty$ and $\mu_n(\partial_\varphi^\lambda) = \sup_{\|T_n(\varphi_m)\|=1} \|\partial_\varphi^\lambda T_n(\varphi_m)\|, n = 1, 2, ...$

In these notations, we have

Theorem. *If for some increasing sequence of natural numbers$\{n_i\}_i^\infty$ the series $\sum_{i=1}^\infty \mu_{n_{i+1}}(\partial_\varphi^\lambda)E_{n_i}(f, \varphi)$ converges.Then $f \in V(\partial_\varphi^\lambda)$* and

$$E_{n_j}(\partial_\varphi^\lambda f, \varphi) \leq 2\sum_{i=j}^{\infty} \mu_{n_{i+1}}(\partial_\varphi^\lambda)E_{n_i}(f, \varphi)$$