MODELING OF CUTTING OF FOOD PRODUCTS

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Abstract: *The mathematical and physical model of cutting of food products is conducted. It is shown out mathematical model which determines dependence of cutting effort on the parameters of regimes of process and structurally mechanical properties of product. Modeling is conducted both for homogeneous products and multi-layered which have a thin, but strong shell, here is a rapid change of cutting effort. At a physical model explored of cutting on the devices of pendulum type, that allows easily to change the parameters of regimes and cutting terms. Certainly cutting effort and rational terms at which spending energy go down and cutting quality rises. Results are applied at development of construction of equipment for cutting of food products.*

Keywords: *cutting, force, mathematical modeling, food products, cutting equipment.*

The food cutting process used in the food industry may serve as a bypassing operation of division of raw material and semi-finished products to pieces of specified shape and size during formation, batching and milling.

Cutting may be as a final operation of product making. The front look and the quality of product surface depend on the quality of its execution. The production profitability of cut and packed products is usually higher in comparison with the rest.

The reference books often lack information on the choices of cutting conditions. That is why the equipment producers pick geometry parameters of working organs of cutting machines and mode conditions of their work in an empirical way. This doesn't lead to the cutting quality improvement and economy of the energy resources.

Food products have various structural and mechanical properties. They perceive the cutting load in different ways. It is known, that when cutting the product is getting deformed prior to destruction. If the comparative deformation of product under the razor face is plastic, but after being cut, the product jams, doesn't restore the previous shape and loses its consumer attractiveness. It can be well examined on the example of cutting fresh hot bread, cheese, salty meat products and many other food samples. If the product is delicate, its destruction is taking place without plastic deformations. During cutting it may collapse into pieces of inaccurate form. It is seen while cutting bread crust, breadcrumbs products, frozen meat and fish. Quality cutting of layered products requires special cutting conditions. They depend on the place of the more firm layer placement in the product. We can take as an example, bread which consists of crumb and crust. Another example – meat in which there are bones and sinewy. Vegetables and fruits that have outward shell. And many packing materials which consist of layers (cardboard, paper, synthetic fiber, skin) which differ in structure. They are not food products but are widely used in the food production industry.

It is known that cutting can be cutoff, cutoff under angel and sliding.

During the cutoff cutting the razor cuts material in standard direction in relation to the cutting bit. During cutoff under angel cutting, the cutting bit is placed under angel in relation to the direction of the razor movement. During sliding cutting the razor embeds into the material in standard direction, making tangent move or product "sliding". To reduce the energy usage for cutting and to provide high quality of surface cut it is necessary to select the conditions of cutting starting from sliding cutting. The continuous contact of razor with the product under high sliding speeds causes considerable friction forces. The quality of surface cut appears low. It is more perspective to use cutoff cutting

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under angle: the product is being cut gradually to a low depth. Cogged lamellate, belt or disk-shaped knives may be used for this.

The analyses of scientific papers showed that the methodologies for directly detecting the power of cutoff cutting is missing and don't let using the gained results during calculation and construction of cutting equipment. The offered method allows to identify dependence of cutting power from razor moving speed and to improve the construction of cutting equipment.

For many viscous, resilient and plastic products the power of cutting is reducing under high razor speeds. The reason for this is that the product under razor edge is being destructed under lower deformation. That is why besides the cutting power decreasing we get a better quality of the cut.

For determination of cutting power we conducted math and physics modeling. This allowed to determine the main parameter of cutting process – the power of cutting in dependence with the razor speed in the product and structural and mechanical property of the product.

Let's take a look at the mechanism of cutting process. Let's compose a differential equation which describes the razor movement in the product. The opposition powers influence the razor (fig. 1).

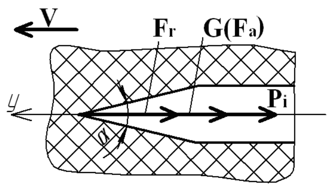


Fig. 1. The scheme of powers that influence the razor.

F_r – cutting power. G – friction power between slide surface of the razor and the product. $F_{aд}$ – adhesion power. P_i – inertia power.

The size of the friction power and adhesion depends on the product characteristics. The friction power is determined under the formula:

$$G = C + k_1 V = C + k_1 \frac{dy}{dt}, \quad (1)$$

C - coefficient, which depends on the load given of the product to the side surface of the knife; *V* – sliding speed between the product and side surface of the knife; k_1 – coefficient of sliding speed influence to the friction power; *y* – razor movement in the product; *t* – razor movement duration in the product.

For the products that stick to the razor, it is necessary to use adhesion power instead of friction power. It depends on the form and state of the razor surface, cutting speed, contact area, cutting angle, structural and mechanical qualities of the product and some other factors.

As function of adhesion strength P_a under ordinary cutting mechanisms tearing off from product surface, power F_a will take down:

$$F_a = P_a S, \quad (2)$$

$P_{aд}$ – adhesion firmness, *Па*; *S* – contacting area, *м*^{2}.

Taking into account that tearing off when cutting is taking place under an angle:

$$F_a = P_a S \cos(\alpha + \alpha_0) \quad (3)$$

α – angle of razor sharpening; α_0 – distinctive angle of tearing off, can be as positive and as negative one.

It is necessary to notice that,

$$0 < \cos(\alpha + \alpha_0) < 1 \quad \Rightarrow \quad 0 < (\alpha + \alpha_0) < 90^o$$

The size of $(\alpha+\alpha_0)$ depends on the form and direction of razor movement, cutting angle.

Inertia power P_i will take down taking into account the direction of razor movement:

$$P_i = ma = m\frac{d^2 y(t)}{dt^2}, \quad (4)$$

m – mass brought to the razor of movable part of the mechanism; *a* – acceleration of razor in the product.

Let's compose the equation of the equilibrium:

$$m\frac{d^2 y(t)}{dt^2} + F_r + F_a = 0 \quad (5)$$

Let's look into most frequent cases of construction and analyses of model (5) for products structural and mechanical qualities of which differ.

Taking into account that firmness of adhesion is linear dependable on the duration of the cutting *t*

$$P_a = b + at, \quad (6)$$

a and *b* are coefficients found by experiment.

We receive the solution to the equation (5). It's important to take into account the starting conditions t=0 =>y=0, dy/dt=V_{0y}, and we receive the equation:

$$y(t) = V_{oy} t - \frac{t^2(3F_r + S\cos(\alpha + \alpha_0)(3b + at))}{6m} \quad (7)$$

Let's differentiate it. Let's define the cutting speed:

$$\frac{dy}{dt} = \frac{2V_{oy}t - 2F_r t - S\cos(\alpha + \alpha_0)(2bt - at^2)}{2m} \quad (8)$$

Let's define the cutting power from equation (8):

$$F_r = \frac{2V_{oy}t - S\cos(\alpha + \alpha_0)(2bt - at^2)}{2t} - \frac{m}{t}\frac{dy}{dt} \quad (9)$$

If dependence of adhesion firmness from cutting time *t* is not linear, then:

$$m\frac{d^2y(t)}{dt^2} + F_r + Be^{bt}S\cos(\alpha + \alpha_0) = 0 \quad (10)$$

$$F_a = Be^{bt}S\cos(\alpha + \alpha_0) \quad (11)$$

$$P_a = Be^{bt} \quad (12)$$

Executing similar actions to the previous case, we find:

$$F_r = \frac{V_{oy}m}{t} - \frac{BS\cos(\alpha + \alpha_0)(b - be^{bt})}{tb^2} - \frac{dy}{dt}\frac{m}{t} \quad (15)$$

Adhesion power can be insignificant, for example, during bread cutting. For bread cutting there is a model known which describes the process. We receive the equation:

$$F_r + G + F_{ad} + P_i = 0 \quad (16)$$

Taking into account equation 1 and 3, open the parts of the equation 16:

$$F_r + (C + k_1\frac{dy(t)}{dt}) + m\frac{d^2y(t)}{dt^2} = 0 \quad (17)$$

Solution in the general view:

$$y(t) = \frac{C_1 \cdot m \cdot e^{-\frac{k_1 \cdot t}{m}}}{k_1} - \frac{(F_r + C)t}{k_1} + C_2, \quad (18)$$

C_1 and C_2 – constants of integration.

Under starting conditions t=0 => $y=0$ => $dy/dt = V_{oy}$ we receive:

$$y(t) = \frac{(F_r + C + V_{oy} \cdot k_1) \cdot m \cdot e^{-\frac{k_1 \cdot t}{m}}}{k_1^2} - \frac{(F_r + C) \cdot t}{k_1} + \frac{(F_r + C + V_{oy}k_1) \cdot m}{k_1^2} \quad (19)$$

Let's differentiate equation 19:

$$\frac{dy(t)}{dt} = \frac{(F_r + C + V_{oy} \cdot K_1) \cdot e^{-\frac{k_1 \cdot t}{m}}}{k_1^2} - \frac{F_r + C}{k_1} \quad (20)$$

From equation (20) we find the cutting power:

$$F_r = \frac{k_1\frac{dy(t)}{dt} - e^{-\frac{k_1 \cdot t}{m}}(C + V_{oy}k_1) + C}{e^{-\frac{k_1 \cdot t}{m}} - 1}, \quad (21)$$

$dy(t)/dt$ - razor movement speed.

Equation 10 can characterize (be the model) the process of cutting of the layered products. In this case the function of exponent e^{bt} should include the negative indicator of degree, for example $b=-a_1$. The indicator of degree $a_1$depends on firmness and location of stronger layer in the mass of the product.

During cutting force and power of cutting equipment calculations it is necessary to know the specific power of cutting as relation to the power of cutting to the length of the cut L.

$$F_{sr} = F_r / L, \text{ Н/м} \quad (22)$$

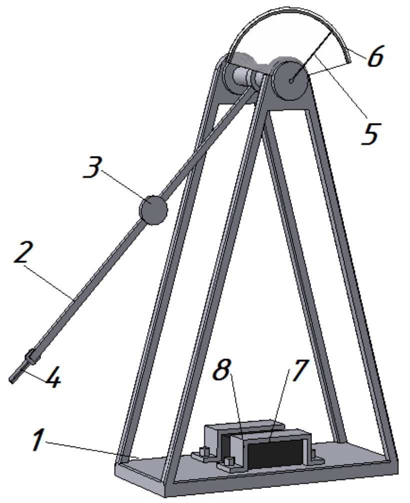
Having experimental meanings of specific power of cutting (from formula 22) the effort of cut off cutting by lamellate flat knife is being determined.

If the knife is cogged, then the effort to the knife is determined by the formula:

$$F_r = F_{sr} \cdot \frac{V_n}{V_t} \cdot H \quad (23)$$

where V_n, V_t - standard and tangensoid knife speeds, H – height of the product's layer.

To determine the cutting power with the usage of received math models we developed experimental rigging. It is simple by construction and safe while using. On the end of the dragonflies 2 razor 4 is placed. Razor during dragonflies falling cuts product 7, which is secured in the fixator 8. Razor speed and its reserve of kinetic energy is easily changed in the wide limits. For this the dragonflies is flinging from various angles, and the placement of load 3 is being changed.



Pic. 2. The scheme of the device for cutting process research:
1 - plate; 2 - dragonflies; 3 - load; 4 - razor; 5 - pointing hand; 6 - scale; 7 - product; 8 – product fixator.

Razor speed:

$$V = R\sqrt{2\frac{\sum P_i r_i}{J}(1-\cos\beta)}\ , \qquad (24)$$

P_i – weight of every detail of dragonflies, r_i – distance from the center of the weight of this detail to the axes of dragonflies; β – angle out of which dragonflies is hurling; R – dragonflies length; J – inertia moment of all dragonflies details.

The received equations and developed methodology of researches allow to define the cutting power in accordance with the speed of razor moving in the material and other characteristics of cutting process; compare structural and mechanical qualities of food products, give assessment of their consistence and quality in accordance with cutting effort.

The results of the modeling are undertaken to define the rational regimes of food products cutting. Cutting power is determined by formula (21). The specific cutting power F_{sr} to the razor length is calculated (formula 22). The results are presented on the fig. 3 and 4.

The results are set in the diapason of razor speeds 1-10 м/с.

Under growth of razor moving speed the specific power of cutting viscid, elastic and plastic products increases first, then reaches the extreme and goes down. It can be seen on the pic. 1 for bread crumb, cheese, not frozen meat.

Reduction of the cutting force is happening at the expense of lessening of product deformation under razor edge under its high speeds. It's typical for all viscid, elastic and plastic products.

For comparatively tight and frail products continuous increasing of cutting force at razor speed growth is observed.

The results gained are valuable for determination of work regimes for cutting equipment. Usage of them in practice allows to lower the energy cutting costs, reduce deformation of products and increase the quality of the cuts.

Peculiarity and novelty of the carried out research is the development of groups of math models. The models allow identifying the actual cutting power for various in its structural and mechanic qualities of products. It was not possible till the present moment because the process was characterized by specific work of cutting and other characteristics, which didn't determine its physical essence.

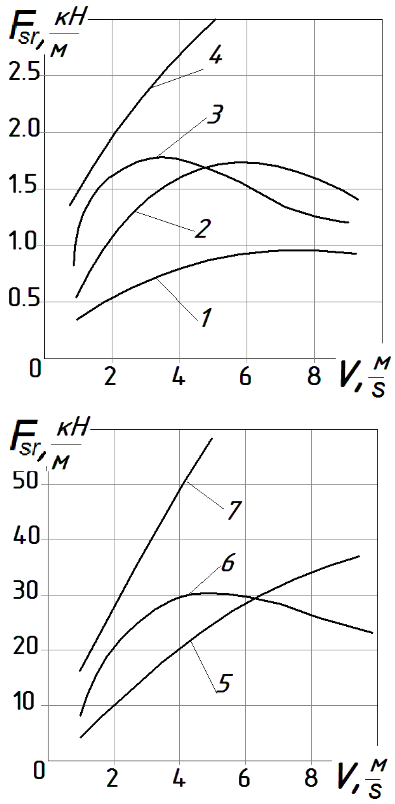


Fig. 3. Dependence of specific cutting power from the razor moving speed in the product during cut off cutting:
1 – crumb of hot bread; 2 – crumb of bread after cooling for 6 hours; 3 – cheese; 4 – sugar beet; 5 – scab bread; 6, 7 – meat (pork) under temperature 5° C and -5°C.

The application of the results of modeling allow optimize the process for many food products. Besides determination of rational cutting regimes with the help of developed methodology it is possible to determine the consistency of the product. The consistency together with other indicators allows evaluate the product quality.

This supports the necessity of further analytical researches and storage of experimental material on this very topic.

References

1. Гуць В.С., Коваль О.А. Адгезия пищевых продуктов в процессах паковки // Упаковка. - 2006. - № 2. С. 39-41.

2. Гуць В.С., Губеня А.А. Методика определения усилий резания пищевых продуктов // Вестник МГУП. -2008. - № 2. С. 102-107.