Investigation of operation of holographic interferometer under phase distortions in probe beam.

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ABSTRACT

Influence of non-stationary phase distortions in a transmitting part of holographic interferometer on quality of formation interference pattern was investigated. Conducted in the work were experiments with holographic interferometer with flexible lightguiding bundles and etalons of roughness as a source of statistical phase distortions. Were simulated the phase distortions of various statistical distributions together with their effect to the contrast of the interference pattern. Considered here is a model of interferometer as a correlator random phase distributions with a signal in the form of ideal interference pattern.
Keywords: holographic interferometer, phase distortions, statistical distributions, interferometer as a correlator.

1.INTRODUCTION

The essence of holographic interferometry is a comparison of the wave-fronts of investigated object in different states. At that, the quality of the wave-fronts of the object is supposed to remain unchangeable during the investigations. However, besides the changes of the state of the object, which are the subject of investigation, there are possible an unexpected wave-front modifications due to atmosphere fluctuations, to dust deposition over the object, to distortions in image-transferring systems etc. Negative factors will impair interference fringes that may have effect on accuracy of measurement. Therefore, it seems the distortions influence to be required. Such an investigation is a subject of the paper.

2.THEORY

2.1. Contrast of interference pattern under phase distortions

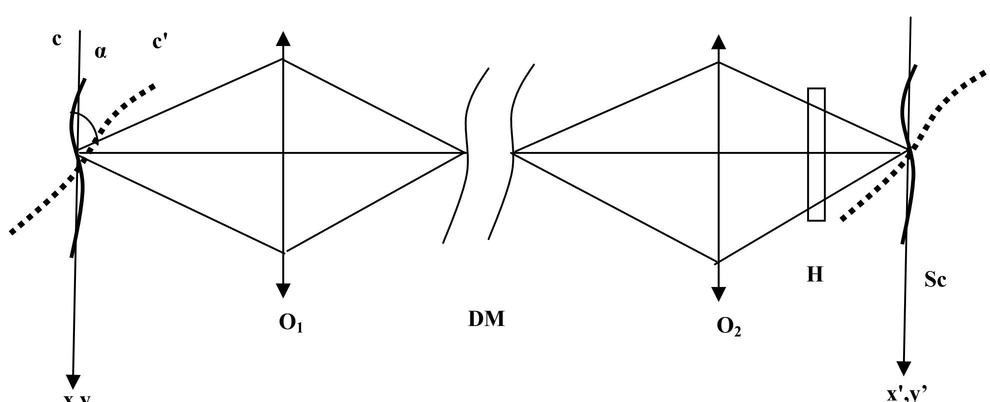

Fig. 1. A scheme of image transfer.

Proposed consideration is based on the following model. Laser radiation in initial plane (x_1,y_1) as show on Fig.1 is being scattered by the surface of the investigated object $A=a(x_1,y_1)\exp[i\varphi(x_1,y_1)]$ in two different states. Assume for determinacy that in the second state the object is turned by a small angle α and its scattered wave-front gets phase addition $\Delta\varphi=\exp[i(\alpha x)]$. The images of the two states of object are being transferred by the optical system $O_1 \ldots O_2$

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through some distorting media **DM** to the image plane $(\mathbf{x_2,y_2})$, recording two holograms in the registering element **H** (see Fig.1). Phase distortion during the first exposure appears as additional phase $\boldsymbol{\psi_1(x_2,y_2)}$, and $\boldsymbol{\psi_2(x_2,y_2)}$ during the second exposure. Under illumination of the recorded hologram an interference field is being formed in the plane $(\mathbf{x_2,y_2})$ with an intensity $\mathbf{I \sim |A_1(x_2,y_2)+A_2(x_2,y_2)|^2}$.
Denote $\psi_1(x_2,y_2)-\psi_2(x_2,y_2)=\psi(x_2,y_2)$ and obtain:

$$I\sim 2|a|^2+|a|^2\exp\{i[\psi(x_2,y_2)-\alpha_2 x_2]\}+|a|^2\exp\{-i[\psi(x_2,y_2)-\alpha_2 x_2]\}= \qquad (1)$$
$$=2|a|^2\{1+\cos[\alpha x_2-\psi(x_2,y_2)]\}$$

This expression describes object luminosity $|a(x_2,y_2)|^2$ modulated with an interference pattern $\{1+\cos[\alpha x_2-\psi(x_2,y_2)]\}$. Now let's analyze the result.
If there is no distorting medium or it is stationary then $\psi=0$, $I_{max}=2$, $I_{min}=0$ and the fringe contrast [1], defined as $V=\frac{\text{Imax}-\text{Imin}}{\text{Imax}+\text{Imin}}$, gets a value of 1 according to the known issues [2].
In the case of non-stationary phase noise $\psi\neq 0$. Generally it is a stochastic function. Let's find an average of the distribution $I(\psi)=\cos(\alpha x-\psi)$, defined as [3]:

$$I_\psi=\langle\cos(\alpha x'-\psi)\rangle=\int_{-\infty}^{+\infty}\rho(\psi)\cos(\alpha x'-\psi)\,d\psi=\text{Re}\int_{-\infty}^{+\infty}\rho(\psi)\,\text{eõð}[i(\alpha x'-\psi)]\,d\psi=$$
$$=\text{Re}\left[\text{eõð}[i\alpha x]\int_{-\infty}^{+\infty}\rho(\psi)\,\text{eõð}[i\psi]\,d\psi\right] \qquad (2)$$

Accordingly fringe contrast function in general case (under phase distortions) is proportional to:

$$V'\sim\int\cos(\psi)\,\rho(\psi)\,d\psi \qquad (3)$$

To evaluate expression (2) we need to determine the distribution density $\boldsymbol{\rho(\psi)}$. Considered in the work were the two most common distributions: Gauss distribution and uniform distribution, also were obtained the functions of contrast of the interference pattern for these distributions:

1. Gauss distribution

$$\rho(\psi)=\frac{1}{\sigma\sqrt{2\pi}}\,\text{eõð}\left(-\frac{\psi^2}{2\sigma^2}\right) \qquad (4)$$

σ – dispertion (standard deviation) of normal distribution.
Substitute (4) into (2) is obtained [4]:

$$I_\psi=\langle\cos(\alpha x'-\psi)\rangle=\frac{\sigma\sqrt{2\pi}}{\sigma\sqrt{2\pi}}\cos(\alpha x')\,\text{eõð}\left(-\frac{\sigma^2}{2}\right)=\cos(\alpha x')\,\text{eõð}\left(-\frac{\sigma^2}{2}\right) \qquad (5)$$

Let's find the maximum and the minimum of the averaged interference field intensity. As follows from (5), the maximum occurs when $\cos(\alpha x')=1$ and the minimum occurs when $\cos(\alpha x')=-1$. Finally, for the fringe contrast under phase noise obtain:

$$\left.\begin{array}{l} I_{max}=1+\text{eõð}\left(-\frac{\sigma^2}{2}\right)\\ I_{min}=1-\text{eõð}\left(-\frac{\sigma^2}{2}\right)\end{array}\right|\Rightarrow V'=\frac{1+\text{eõð}\left(-\frac{\sigma^2}{2}\right)-1+\text{eõð}\left(-\frac{\sigma^2}{2}\right)}{1+\text{eõð}\left(-\frac{\sigma^2}{2}\right)+1-\text{eõð}\left(-\frac{\sigma^2}{2}\right)}=\text{eõð}\left(-\frac{\sigma^2}{2}\right) \qquad (6)$$

A plot of fringe contrast against mean-square deviation $V'(\sigma)$ is shown at the left of Fig.2. At the right of Fig.1 a series of computer simulated images of interference pattern for certain points on the plot are depicted.

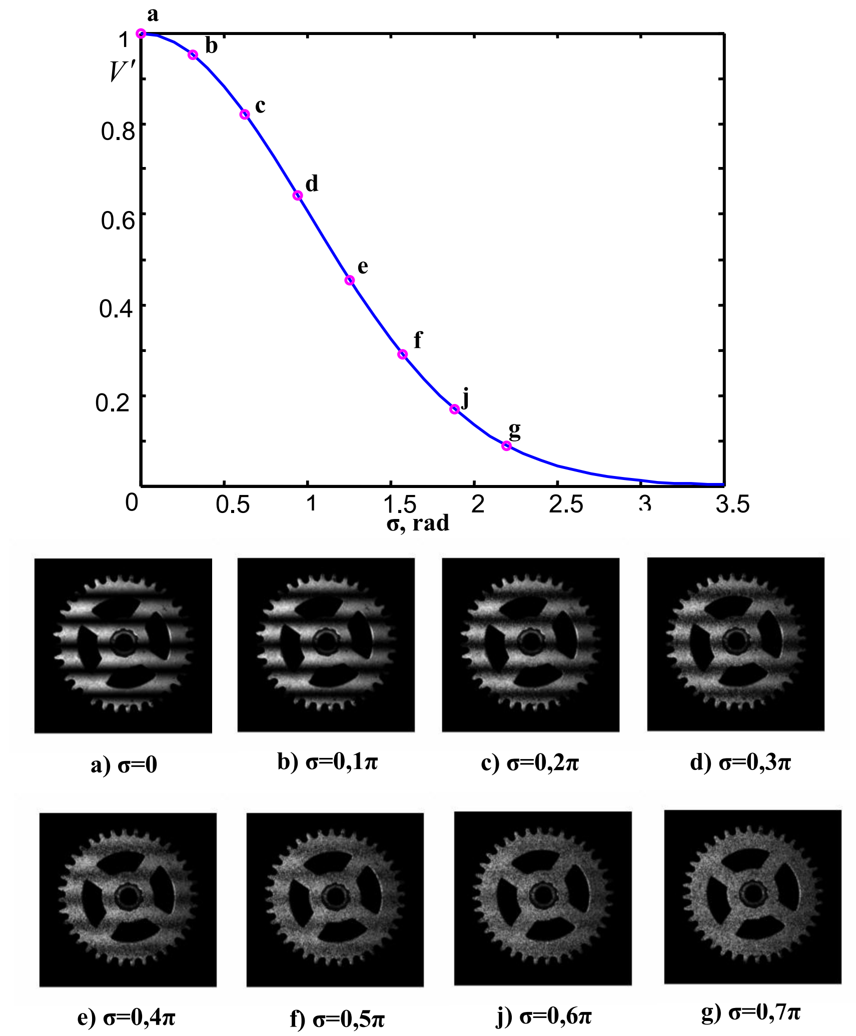


Fig.2. A plot of fringe contrast *V'* against σ and views of interferograms for certain values of σ.

2. Uniform distribution

$$\rho(\psi)=\left\{\begin{array}{l}\psi > a \Rightarrow \rho(\psi)=0 \\ \psi < a \Rightarrow \rho(\psi)=\dfrac{1}{2a}\end{array}\right\} \qquad (7)$$

where **a** – halfwidth of uniform distribution.
Substituting the obtained formula (7) into (2) we get:

$$I_{\psi}=\mathrm{Re}\left[\exp[i\alpha x']\frac{1}{2a}\int_{-a}^{+a}\exp[-i\psi]\,d\psi\right]=\ldots=\cos(\alpha x')\,\mathrm{sinc}(a) \qquad (8)$$

Considering the formula (6) and the above thinking the fringe contrast function for uniform distribution will look like:

$$\left.\begin{array}{l} I_{max}(\cos(\alpha x')=1)=1+\mathrm{sinc}(a) \\ I_{min}(\cos(\alpha x')=-1)=1-\mathrm{sinc}(a) \end{array}\right| \Rightarrow V'=\frac{1+\mathrm{sinc}(a)-1+\mathrm{sinc}(a)}{1+\mathrm{sinc}(a)+1-\mathrm{sinc}(a)}=\mathrm{sinc}(a) \tag{9}$$

The dependence of fringe contrast vs. the halfwidth of the distribution, plot following the formula (9), is shown in the

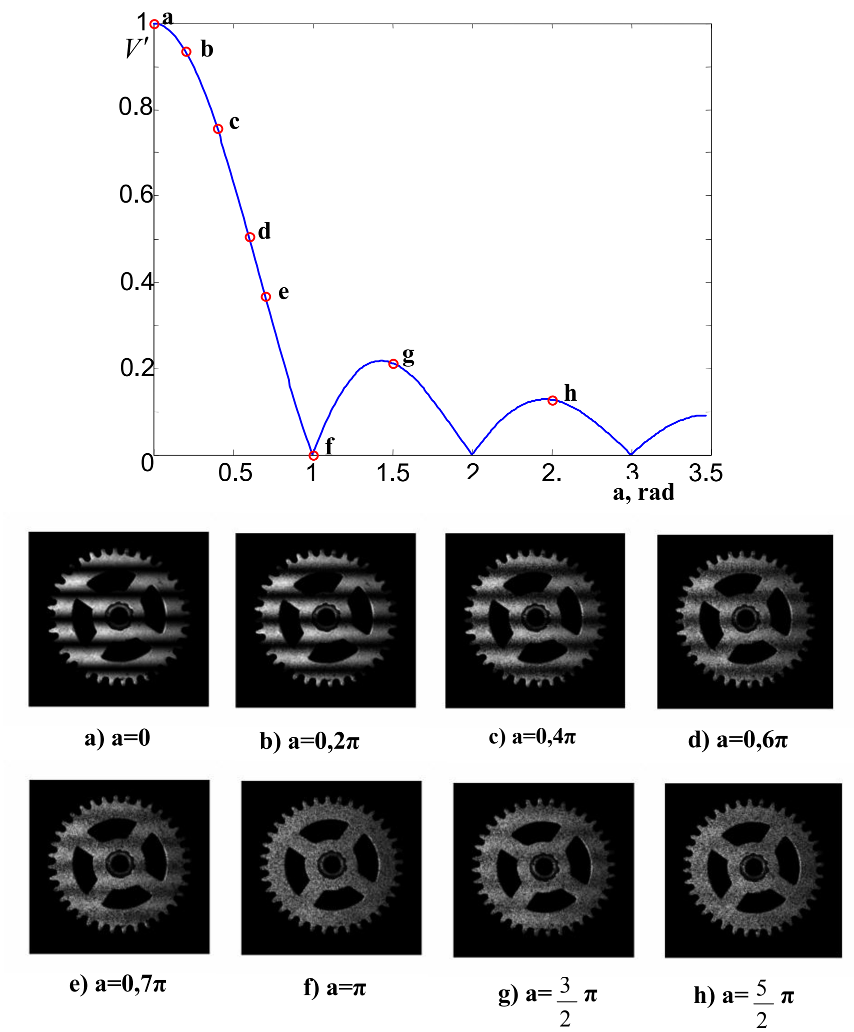


Fig.3. A plot of fringe contrast *V'* against *a* and views of interferograms for certain values of σ.

upper part of Fig.3. In the lower part of Fig.3. shown is the images of the interference pattern simulated on the PC for certain points of the plot.

Scientifically and practically important results were obtained upon modeling of the situation, where the phase noise is described with the same type of distribution but with different dispersion in different parts of the object. For example, on Fig.4. shown are the images of interference pattern for normal distribution with different dispersion in the left and right parts of the object. Let's consider image 9) of the Fig.4. One can see from the image that the intereference fringes in left and right parts reveal the shift against each other in half of period. It takes place when the values of the dispesion in different parts of the image lay at opposite sides with respect to the sign change point of the Sinc function (a1=0,6π and a2=3/2 π, while thw sign change point is a=π). This is due to solely the statistics of the microscopic scatterers and may be a source of artifacts in deformation measurements and thus should be kept in mind, especially, when the distribution is more complicated than (6).

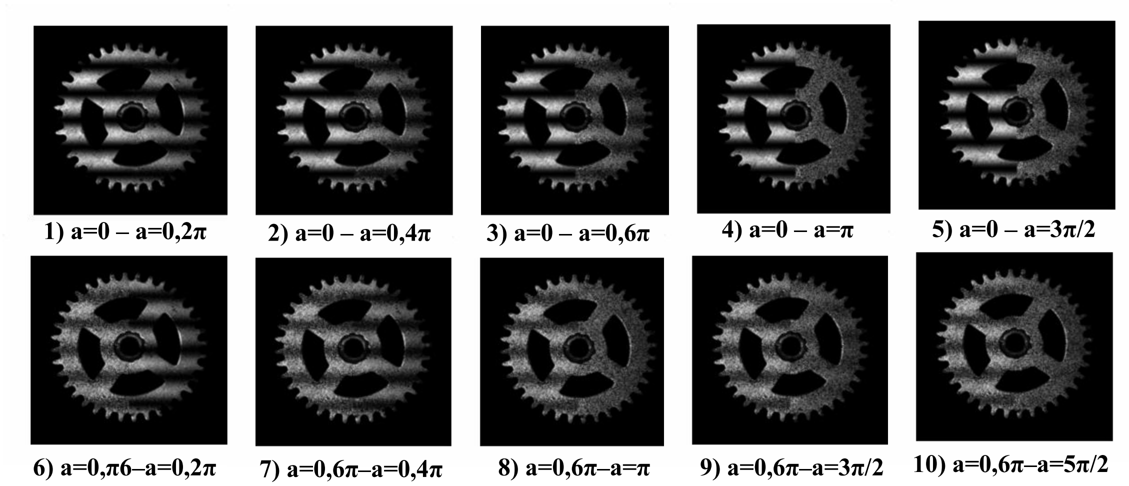


Fig.4 Interferograms with different *a* value in different parts of the object.

The distribution function may be obtained from experiment by means of scanning over the dispersion and measuring the fringe contrast. The implementation of such an experiment is a subject for a separate work.

2.2 A correlator of phase distortions with response in the form of interference pattern.

We have investigated the effect of the phase noise statistics on the fringe contrast. However, the inverse problem is also definable, i.e. in which way the measurable fringe contrast determines the measure of microscopic distortions in interferometric investigation. Due to this we consider the following experiments on definition of spatial correlation of the two rough surfaces on the holographic correlator.

Theoretical consideration, numerical simulation and experimental researches were carried out according to the classical scheme of recording Fourier hologram, but at its correlation reconstructing (by signal beam). The scheme of recording and restoration of Fourier hologram is reduced to equivalent telescopic system [2]

The mathematics of the operation of the scheme is the following. The complex amplitude of the wave restored in +1 diffraction order in the plane just after the hologram is described as [6]:

$$r_{+1} = F^{-1}\left\{A^{*}BC\right\} = \left(\tilde{a} * \tilde{c}\right) \otimes \grave{a}_2 \tag{9}$$

it determines spatial cross-correlation function of the tow used diffusers $\tilde{a}$ and $\tilde{n}$ with intersection *S.*
Let's write down (9) in an obvious form:

$$r_{+1} = a_1 \grave{a}_2 a_3 \iint_S \exp\left[i(d - d - \psi)\right] dx\,dy = a_1 a_2 a_3 \iint_S \exp\left[-i\psi\right] dx\,dy \tag{10}$$

and consider the value of the integral.

$$\iint_S \exp\left[-i\psi\right] dx\,dy = \iint_S \left(\cos(\psi) - i\sin(\psi)\right) dx\,dy = \iint_S \left(\cos(\psi)\right) dx\,dy - \iint_S \left(i\sin(\psi)\right) dx\,dy \tag{11}$$

Let's analyze all over again the second term in the formula (11). The Sin function is odd and its argument is a random variable with zero average value. In limits of integral the argument gets values above and below zero in identical quantity. As consequence $\iint_S \left(i\sin(\psi)\right) dx\,dy = 0$

Let's pay attention to the first term in the formula (11). If we divide it by the area of diffuser *S* it will determine an average value of function cos *(ψ)* [4]. On the other hand average value of cos *(ψ)* is defined as:

$\int \cos(\psi)\,\rho(\psi)\,d\psi = M(\cos(\psi))$ [4], where *M (*cos *(ψ)*) is an average value of cos *(ψ)*, and $\rho(\psi)$ - distribution density of phase incursion.

We have *(V' ~* $\int \cos(\psi)\,\rho(\psi)\,d\psi$ *)* (3)*,* that is *E* ~ *V '.* And as $I = |E|^2$, there is clear that fact, that

$$I \sim V'^2 \tag{12}$$

3. EXPERIMENT

In this part of the work we compare experimental results with theoretical data, for that used holographic interferometer scheme with the standard phase distortions object. This experimental is complimentary to theoretical part about contrast of interference pattern under phase distortions. Object is mirror of the 13th class of accuracy of surface treatment that is shifted in the plane of object (as shown in Fig.5) between the two exposures of hologram for the compared object states. At that in the second state the object is turned by a small angle **α**, as had been considered in the theory.

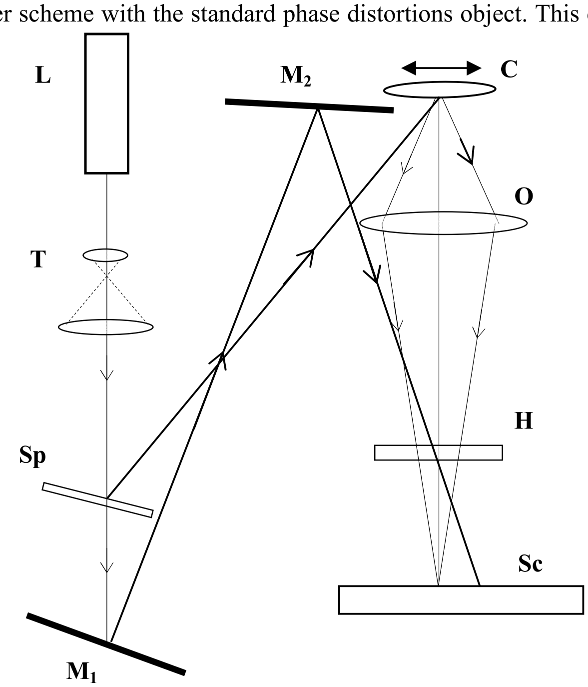


Fig.5.**The setup scheme.**
L – the He-Ne laser; T – the telescope, Sp – the glass plate; M_1 та M_2 – the mirrors; O– the objective; C – the object, H – hologram; Sc – the screen.

Assuming distribution density of the surface irregularity to be Gaussian we can express σ (standard deviation) through R_a (mean deviation of absolute value) **σ=R$_a$*π**. (R_a(13thclass)=0,016mkm). Standard deviation of phase distortions between the two exposures of hologram (due to object shift) is **σ_Σ=2σ** [4]. Thus one obtains theoretical value of standard deviation for object of the 13th class of accuracy of surface treatment **σ_Σ=0,3π** and according the fringe contrast (from Fig.2) is **V'=0,62**

On the Fig.6a are shown the photograph of the interferogram and the plot of intensity distribution for the case of stationary process, fringe contrast is **V'=0,8**. On the Fig.6b are shown the experimental results for the case of non-stationary process **(σ_Σ=0,3π),** fringe contrast is **V'=0,5.**

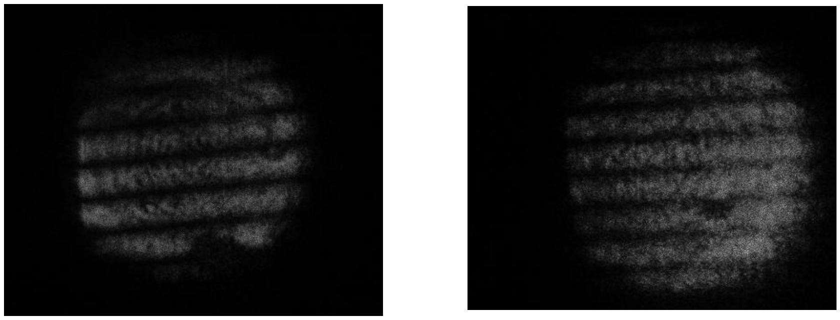
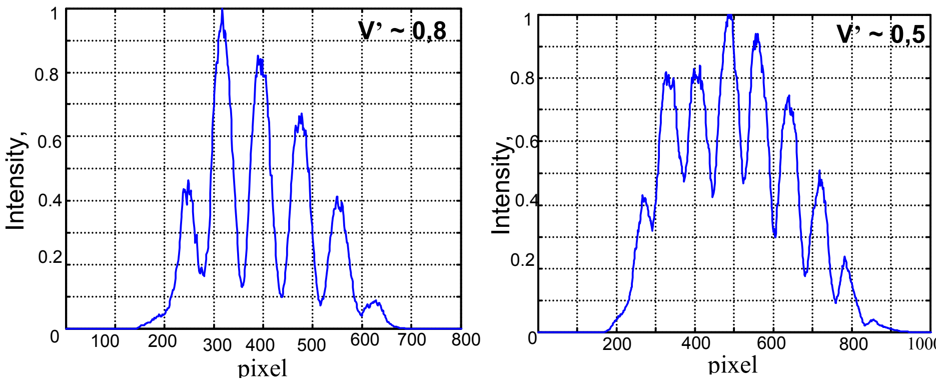


Fig6 a) interferogram without phase noise and **b)** interferogram with phase noise **(σ_Σ=0,3π)** and correspondingly the plots of the normalized intensity averaged along the interference fringes.

a) **b)**



On **fig 6** represented is optical scheme of experiment complimentary to theoretical part about the correlator of phase distortions with response in the form of interference pattern.

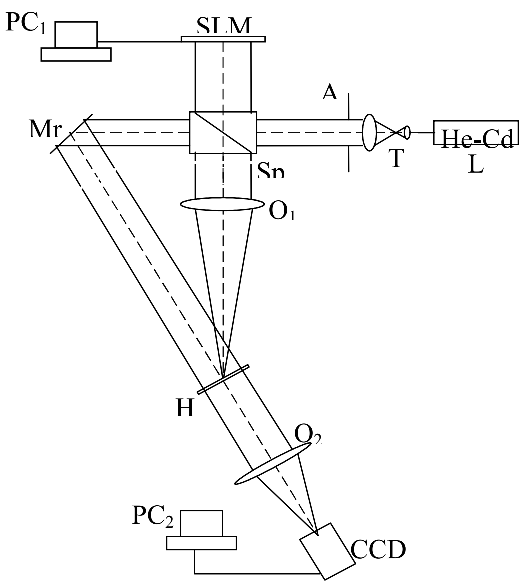


Fig 7 The scheme of optical-digital correlator
L- He-Cd laser (λ=441,6 nm), T- telescope, A – aperture, Sp - beam spliter, Mr – revolving mirror, H – hologram, O_1 and O_2- Fourier objective, SLM - spatial light modulator,

Process of obtaining of experimental results consist of the following stages:

1. Calculation. On a computer in program MatLab the initial diffuser is calculated. Distributions of a phase in all the next diffusers are a compositions of distributions of initial diffusera and random phase distortions with predefined distribution.

2. Experiment. Registered was on optical-digital correlator (fig. 3) the intensity of correlation peak between initial diffuser and diffusers with a controllable dispersion. For this on SLM (LC-Rr500, reflective, grayscale, resolution-1024x768, bit-depth-8 bit) distribution of a phase of initial diffuser was produced. Angular spectrum of diffuser on hologram H was recorded. Hologram is photopolymer (FPK-488), with diffraction efficiency 55%. Reference to object beam ration is 10:1. Further on SLM were consistently produced diffusers with adjusted dispersion. After every new reconstructing diffuser fixed was intensity of correlation peak between diffuser on SLM and initial diffuser recorded on hologram H.

3. Processing results. In program MatLab was plot the dependence of intensity correlation peak, received on experiment, either against the dispersion (for Gauss distribution), or against half- width of distribution function for phase distortion (for uniform distribution) of corresponding diffuser. The received dependence was compared to function of contrast of interference fringes for corresponding distributions (4), (7). The result is presented on fig. 8.

On fig 8b is represented the dependence of intensity of correlation peak for diffusers calculated with uniform

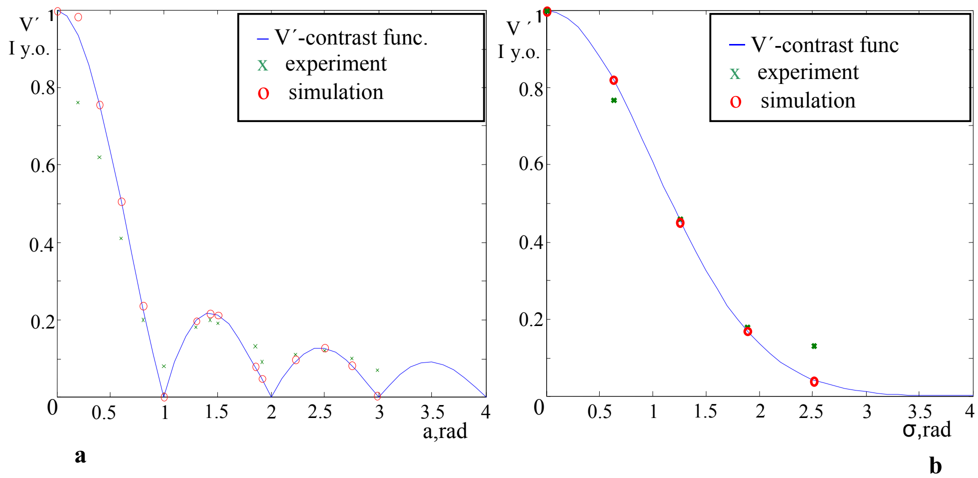


Fig. 8 The dependence of interference pattern contrast *V'* and of correlation peak intensity I on the scale of phase noise for **a)** uniform noise (a – the width of the distribution function) and for **b)** Gaussian noise (σ – the dispersion of the distribution)

distribution and corresponding function of contrast *V´* (7). And on fig. 8a is represented the dependence of intensity of correlation peak for diffusers calculated with Gauss distribution, and corresponding function of contrast *V´* (4).

For comparison on fig. 8 are shown the results of the simulation intended for some kind of graduation of real experiment. Stages of simulation experiment were analogous to the real one. But difference was that the second stage was conducted with the help of the PC. For that the resulted of a theoretical part of this work were used.

4. CONCLUSIONS

In the work proposed is the theoretical data establishing a connection between the conditions of hologram recording, statistical parameters of distortion medium and contrast of interference fringes due to the test modification of the object location. Also shown is the interferogram of the object for the case of stationary and non-stationary (σ_Σ=0,3π) process. Fringe contrast obtained from experiment for the case of stationary process is V'~ 0,8 (less than 1). It may be explained due to different intensities of interfering beams [5]. We have shown that spatial distribution of a field in interference pattern can be treated as convolution of a clear fringe pattern with correlation function of spatial distributions of phase distortions on a surface of the object. Accordingly the interferometer itself is treated as correlator of statistical fields with a signal in the form of the interference pattern that describes the changes of the form of the object.

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