ON SOME ESTIMATES OF SINGULAR NUMBER OF INTEGRAL OPERATOR OF HILBERT-SCHMIDT

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Let be A an integral operator of Hilbert-Schmidt with kernel $a(t,s)$ acting in the space $L_2[0;1]$ and let $s_k(A)$ its singular values. We establish that for all $r = 2, 3, ...$ the estimate

$$\sum_{k=r}^{\infty} s_k^2(A) \leq 2\omega^2 \left(\frac{1}{r-1}, a\right)_2,$$

holds. Here $\omega(\delta, a)_2$ is the modulus of continuity of the kernel $a(t,s)$ given by the formulas

$$\omega(\delta, a)_2 := \sup_{0\leq \ h\leq\delta} \left(\int_0^1 \int_0^{1-h} |a(t+h,s) - a(t,s)|^2 \, dt \, ds \right)^{\frac{1}{2}}, \quad 0 < \delta \leq 1$$

Also we have established similar estimates in terms of the modulus of continuity more then one order in the case, when domain of image of the operator A belongs to the space of continuity functions.

For all $m = 2, 3, ...$ the estimate

$$\sum_{k=r}^{\infty} s_k^2(A) \leq 25\omega_m^2 \left(\frac{2}{r}, a\right), \quad r = m+1, m+2, ... \tag{1}$$

holds. Here $\omega_m(\delta, a)$ is the modulus of continuity of the kernel $a(t,s)$ given by the formulas

$$\omega_m(\delta, a) := \sup_{f\in L_2, \|f\|_2 = 1} \ \sup_{0\leq \ h\leq\delta} \ \sup_{0\leq \ t\leq 1-hm} |\sum_{q=0}^{m} (-1)^q C_m^q (Af)(t+hq)|.$$

The formula (1) yields the following proposition. Let the domain of values of the operator A belong to a space of continuous functions, and $m = 2, 3, ...$

$$s_r(A) \leq \left(\frac{10m}{r}\right)^{\frac{1}{2}} \omega_m \left(\frac{4}{r}, a\right), \quad r = m+1, m+2, \ldots$$

1. I. Fredholm, Sur une classe dequations fonctionnelles, Acta Math., 27, 365390 (1903).

2. I. C. Gohberg and M. G. Krein, Introduction to the Theory of Linear Nonself-Adjoint Operators, Amer. Math. Soc., Providence, RI, 1969.