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ARCHED TRANSMISSIONS TEETH GEOMETRY WITHIN OFFSET OF INITIAL PROFILE

There has been researched teeth geometry of cylindrical arched teeth gears, cut with rod instrument, teeth of which profiled by arbitrary curve in normal cross-section and in linear direction within initial profile offset. Toothing of instrumental rod with arched teeth with gears within initial profile offset is an analogue of the process of cutting teeth with rod instrument by means of enveloping. Surfaces of gear teeth within toothing with rod are enveloping of teeth surfaces of the last one. If teeth surfaces of instrumental rods for cutting pinion teeth and gear are non-congruous, then we have case of dotty toothing of pinion and gear. If initial profiles are non-congruous, then contact dot moves from one side of the tooth to another. It was obtained equations of teeth surfaces, while writing down coordinates of toothing surfaces in coordinate systems $X_1Y_1Z_1$ and $X_2Y_2Z_2$, connected with pinion and gear. While making such a transition, we have equations of surfaces of convex side of pinion teeth and concave side of gear teeth. In common case those equations are equations of helical lines of variable pitch. These equations can be used within the determination of toothing field borders, appropriate to the tops of the pinion and the gear teeth. Within this case two types of toothed gears with arched teeth will differ: gears with symmetric arched teeth (teeth are symmetric relatively to the plane XOZ and asymmetric arched teeth (working area of teeth surface is situated from one side of plane). Received results can be used within the determination of indices of loading ability and other characteristics of cylindrical arched transmissions with generalized teeth geometry within initial profile offset.

***Key words**: initial profile; producing surface; arched transmissions; profile offset; instrumental rod toothing.*

Introduction. Modern terms of market economy raise the tasks of quality increase, reliability and durability of cars and mechanisms in front of enterprises of machine-building. Teeth transmissions take one of the leading places among the output of machine-building branch because they are the constituent part of drives of practically of all cars. That is why the increase of qualitative indices of teeth toothing by means of the perfection of transmission teeth geometry is an actual scientific and technical task.

Geometrical and kinematic indices of loading transmission ability in dependence of the unknown functions, determining cutting instrument geometry of rod type, are necessary within the synthesis of teeth geometry of teeth gears [1]. General questions of plane toothings' geometry are considered in works [2, 3, 4]. Though obtained in it results and relations do not allow to produce synthesis of cylindrical teeth gears' geometry according to loading ability indices. In works [3, 4] geometry of cylindrical arched teeth transmission, formed by means of generalized producing surface, has been researched. Though, research data cannot be applied for arched transmission within the presence of initial profile.

Objects and problems. Generalized surface of arched teeth of instrumental rod within the initial profile offset.

Let us consider coordinate system $X_n Y_n Z_n$ (Fig. 1). Curve $\bar{r}_0(\mu)$, determining linear teeth form of producing surface has been given in coordinate system $X_n Y_n Z_n$, and rod teeth profile is drawn by generalized initial profile in normal cross-section.

Let us present the equation of teeth surface of rod instrument (producing surface) in connected with it coordinate system in the form of vector [1] in order to produce geometry synthesis of cylindrical teeth gears according to indices of loading ability

$$\bar{r}_n = \bar{r}_0(\mu) + \bar{b}_0 f_1(\lambda) + \bar{n}_0 f_2(\lambda), \qquad (1)$$

where $\bar{r}_0(\mu)$ – vector, determining linear teeth form of producing surface; $\bar{b}_0$, $\bar{n}_0$ – single vectors of binormal and curve normal $\bar{r}_0(\mu)$; $f_1(\lambda)$, $f_2(\lambda)$ – functions, determining initial profile geometry of cutting rod instrument (producing surface) in normal cross-section ; λ, μ – independent parameters.

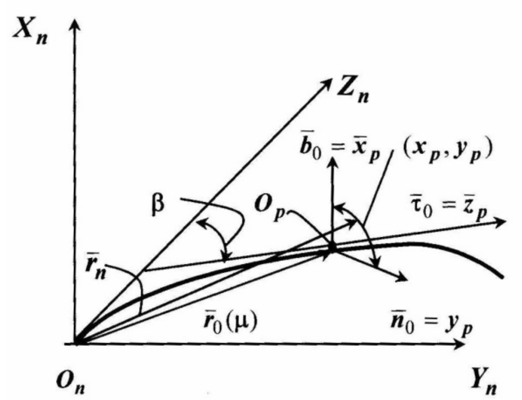

Fig. 1. Producing surface of arched teeth (teeth surface of instrumental rod)

While using the results of the work [3, 4, 5], we obtain equations of arched teeth surface of instrumental rod in the following form:

- convex side of arched teeth (initial profile profile is placed above initial straight line) (Fig. 2)

$$\begin{aligned} x_n &= f_1 + \xi, \\ y_n &= y_0 + f_2 \cos\beta, \\ z_n &= z_0 - f_2 \sin\beta; \end{aligned} \qquad (2)$$

- concave side of arched teeth (initial profile profile is placed above initial straight line within $m = 1\,mm$) (Fig. 3)

$$\begin{aligned} x_n &= f_1 + \xi, \\ y_n &= y_0 - f_2 \cos\beta + 0{,}5\pi, \\ z_n &= z_0 + f_2 \sin\beta, \end{aligned} \qquad (3)$$

where y_0, z_0 – vector projection on coordinate axis; β – the corner of teeth incline (the corner between axis O_nZ_n and tangent to curve (Fig. 1), determined from relations: $\sin\beta = \frac{\dot{y}_0}{\sqrt{(\dot{y}_0^2 + \dot{z}_0^2)}}$, $\cos\beta = \frac{\dot{z}_0}{\sqrt{(\dot{y}_0^2 + \dot{z}_0^2)}}$; $\dot{y}_0$, $\dot{z}_0$ – function derivatives y_0, z_0 in μ; ξ – initial profile offset.

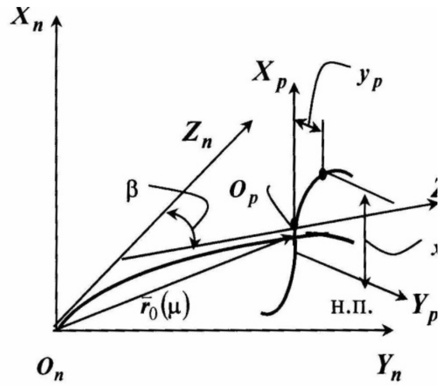

Fig. 2. Convex side of arched tooth of instrumental rod

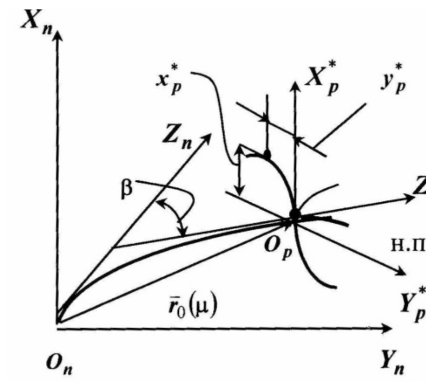

Fig. 3. Concave tooth side of instrumental rod

Toothing of instrumental rod with arched teeth with gears within initial profile offset.

This toothing is an analogue of the process of cutting teeth with rod instrument by means of enveloping. Surfaces of gear teeth within toothing with rod are enveloping of teeth surfaces of the last one.

The scheme of instrumental rod toothing with gears is presented at fig. 4.

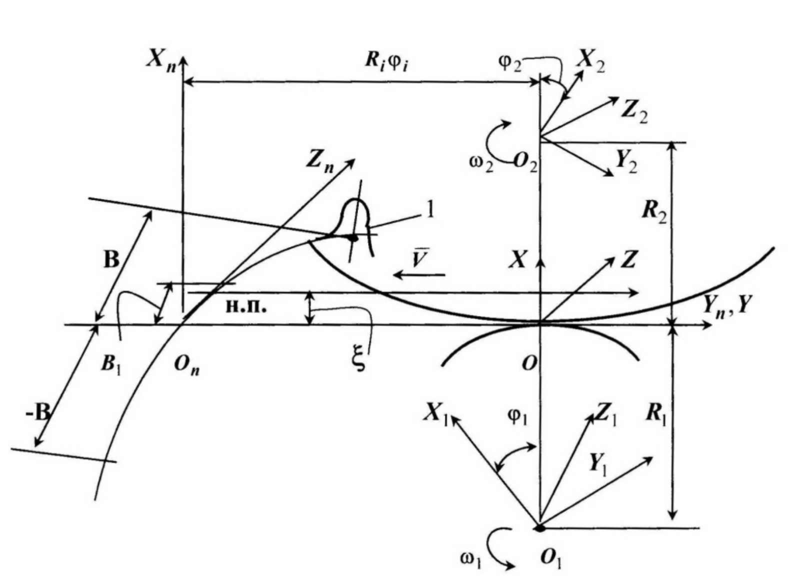

Fig. 4 Scheme of instrumental rod toothing with pinion and gear
(1 – the profile of rod arc tooth in normal cross-section)

Here O_1 and O_2 – are axes of pinion and gear, R_1, R_2 – are radiuses of initial cylinders of pinion and gear, IP – initial rod plane; $X_nY_nZ_n$ – coordinate system, connected with rod; $X_1Y_1Z_1$ – coordinate system, connected with pinion (less gear of teeth pair); $X_2Y_2Z_2$ – coordinate system, connected with gear(larger gear of teeth pair); coordinate system, connected with gear (large gear of tooth pair); XYZ – unmovable system of coordinates. Plane $Y_nO_nZ_n$ – initial rod plane ; plane YOZ – is in the initial rod plane; axes O_1Z_1, O_2Z_2 – are directed along gear axes; axis OZ coincides with pole toothing straight line; φ_1, φ_2 – the corners of pinion and gear rotation. ($\varphi_1 = u\varphi_2$, where u – transfer number of tooth transmission); $\overline{V}$ – linear rod speed ($V = \omega_1 R_1 = \omega_2 R_2$); ω_1, ω_2 – corner speeds of pinion and gear.

Within gear rotation (pinion rotation) on the corner φ_i ($i = 1$ – for the pinion, $i = 2$ – for the gear) rod will offset on the distance $R_i\varphi_i$.

Equations of surfaces of arched rod teeth in unmovable coordinate system XYZ have the following form:

- for left profile of initial profile (convex side of arched teeth) with the usage (2)

$$\begin{aligned} x &= f_1 + \xi, \\ y &= y_0 + f_2 \cos\beta - R_i\varphi_i, \\ z &= z_0 - f_2 \sin\beta; \end{aligned} \qquad (4)$$

- for right profile of initial profile (concave side of arched teeth $\boldsymbol{m} = 1$ mm) with the usage (3)

$$\begin{aligned} x^* &= f_1 + \xi, \\ y^* &= y_0 - f_2 \cos\beta + \pi/2 - R_i\varphi_i, \\ z^* &= z_0 + f_2 \sin\beta. \end{aligned} \qquad (5)$$

Equations of rod toothings with teeth gears we will write down in the form [2]:

- within rod and pinion toothing

$$F_1^* = \overline{e} \cdot \overline{V}^{p1}; \qquad (6)$$

- within rod and gear toothing

$$F_2^* = \overline{e} \cdot \overline{V}^{p2}, \qquad (7)$$

where $\overline{V}^{p1}$, $\overline{V}^{p2}$ – vectors of relative speeds within rod toothing with pinion and gear;

$\overline{e}$ – single vector of normal with coordinates:

$$l_{xn}=\frac{\pm f_2'}{n},\quad l_{yn}=-\frac{f_1'}{n}\cos\beta,\quad l_{zn}=\frac{f_1'}{n}\sin\beta, \tag{8}$$

where $n=\sqrt{(f_1')^2+(f_2')^2}$;

Vectors of relative speeds are equal within $\omega_1=1\frac{1}{c}$, $\omega_2=1\frac{1}{c}$

- within rod toothing with pinion and gear with the usage (4) (convex side of arc pinion teeth, concave side of gear arc teeth)

$$\begin{aligned}\overline{V}^{p1}&=-(y_0+f_2\cos\beta-R_1\varphi_1)\bar{i}+(f_1+\xi)\bar{j}+o\cdot\bar{k},\\ \overline{V}^{p2}&=(y_0+f_2\cos\beta-R_2\varphi_2)i-(f_1+\xi)\bar{j}+o\cdot\bar{k};\end{aligned} \tag{9}$$

- within rod toothing with pinion and gear with the usage (5) (concave side of arc pinion teeth, convex side of arc gear teeth)

$$\begin{aligned}\overline{V}^{p1}&=-(y_0-f_2\cos\beta+\pi/2-R_1\varphi_1)\bar{i}+(f_1+\xi)\bar{j}+o\cdot\bar{k},\\ \overline{V}^{p2}&=(y_0-f_2\cos\beta+\pi/2-R_2\varphi_2)\bar{i}-(f_1+\xi)\bar{j}+o\cdot\bar{k},\end{aligned} \tag{10}$$

where $\bar{i}$, $\bar{j}$, $\bar{k}$ – unit vector of unmovable coordinate system.

It is necessary to multiply ω_1 and ω_2 appropriately for identifying true meanings of vector projections of relative speeds $\overline{V}^{p1}$ and $\overline{V}^{p2}$.

Let us present equations of toothing (6) and (7) with the account of meanings of vector $\bar{e}$ projections in the following way:

- within rod toothing with pinion and gear (convex side of pinion teeth, concave – of gear teeth)

$$\begin{aligned}F_1^*&=-(y_0+f_2\cos\beta-R_1\varphi_1)\cdot\frac{f_2'}{n}-\frac{(f_1+\xi)f_1'}{n}\cos\beta=0,\\ F_2^*&=(y_0+f_2\cos\beta-R_2\varphi_2)\cdot\frac{f_2'}{n}+\frac{(f_1+\xi)f_1'}{n}\cos\beta=0;\end{aligned} \tag{11}$$

- within rod toothing with pinion and gear (concave side of pinion teeth, convex – gear teeth)

$$\begin{aligned}F_1^*&=-(y_0-f_2\cos\beta+\frac{\pi}{2}-R_1\varphi_1)\cdot\left(-\frac{f_2'}{n}\right)-\frac{f_1'(f_1+\xi)}{n}\cos\beta=0,\\ F_2^*&=(y_0-f_2\cos\beta+\frac{\pi}{2}-R_2\varphi_2)\cdot\left(-\frac{f_2'}{n}\right)+\frac{f_1'(f_1+\xi)}{n}\cos\beta=0.\end{aligned} \tag{12}$$

It follows from (9), (10), (11), (12)

$$\overline{V}^{p1} = (\Omega_1 \cos\beta)\bar{i} + (f_1 + \xi)\bar{j} + o \cdot \bar{k},$$
$$\overline{V}^{p2} = (-\Omega_1 \cos\beta)\bar{i} - (f_1 + \xi)j + o \cdot \bar{k}, \qquad (13)$$

where $\Omega_1 = \frac{(f_1 + \xi)f_1'}{f_2'}$.

Equations (11) and (12) are additional terms of parameter connection λ, μ, φ_i. Equations of surfaces of teeth gear and rod toothing in unmovable coordinate system XYZ (as well as toothing of pinion teeth and gear) with the usage of (11), (12) and (4), (5) can be written down:

- within toothing of convex side of pinion and concave – gear teeth

$$x = f_1 + \xi,$$
$$y = -\Omega_1 \cos\beta,$$
$$z = z_0 - f_2 \sin\beta; \qquad (14)$$

- within toothing of concave side of pinion teeth and convex – gear teeth

$$x = f_1 + \xi,$$
$$y = \Omega_1 \cos\beta,$$
$$z = z_0 + f_2 \sin\beta. \qquad (15)$$

Within $\varphi_i = \text{const}$ equations (14), (15) determine immediate teeth line contact on the surface of toothing. Within $z = \text{const}$ the first two equations (14), (15) determine toothing line in side gear plane. Within $\mu = \text{const}$ we obtain equations of line toothing in normal cross-section of toothing gears. If teeth surfaces of instrumental rods for cutting pinion teeth and gear are non-congruous, then we have case of dotty toothing of pinion and gear. If initial profiles are non-congruous, then contact dot moves from one side of the tooth to another. ($\lambda = \text{const}$ within it.) If rod surfaces along teeth length are non-congruous, then contact dot moves along teeth height ($\mu = \text{const}$). We have an analogue of Novikov's toothing in the first case, localization of pinion teeth contact and gear along their length.

Surface geometry of pinion arc teeth and gear within initial profile offset

We will obtain equations of teeth surfaces, while writing down coordinates of toothing surfaces (14) and (15) in coordinate systems $\boldsymbol{X}_1\boldsymbol{Y}_1\boldsymbol{Z}_1$ and $\boldsymbol{X}_2\boldsymbol{Y}_2\boldsymbol{Z}_2$ (fig. 4), connected with pinion and gear. While making such a transition, we have:

- equations of surfaces of convex side of pinion teeth and concave side of gear teeth (while using equations (14)

$$x_1 = (f_1 + \xi + R_1)\cos\varphi_1 + \Omega_1 \cos\beta \sin\varphi_1,$$
$$y_1 = (f_1 + \xi + R_1)\sin\varphi_1 - \Omega_1 \cos\beta \cos\varphi_1,$$
$$z_1 = z_0 - f_2 \sin\beta; \qquad (16)$$

$$x_2 = (f_1 + \xi - R_2)\cos\varphi_2 - \Omega_1 \cos\beta \sin\varphi_2 ,$$
$$y_2 = -(f_1 + \xi - R_2)\sin\varphi_2 - \Omega_1 \cos\beta \sin\varphi_2 ,$$
$$z_2 = z_0 - f_2 \sin\beta; \qquad (17)$$

- equations of surfaces of concave side of pinion teeth and convex side of gear teeth (while using equations (15)

$$x_1 = (f_1 + \xi + R_1)\cos\varphi_1 - \Omega_1 \cos\beta \sin\varphi_1 ,$$
$$y_1 = (f_1 + \xi + R_1)\sin\varphi_1 + \Omega_1 \cos\beta \cos\varphi_1 ,$$
$$z_1 = z_0 + f_2 \sin\beta; \qquad (18)$$

$$x_2 = (f_1 + \xi - R_2)\cos\varphi_2 + \Omega_1 \cos\beta \sin\varphi_2 ,$$
$$y_2 = -(f_1 + \xi - R_2)\sin\varphi_2 + \Omega_1 \cos\beta \sin\varphi_2 ,$$
$$z_2 = z_0 + f_2 \sin\beta. \qquad (19)$$

In equations (16) – (19) variables λ μ, φ_1, φ_2 are connected with relations (11), (12). Profiles of pinion teeth and gear in normal cross-section are identified with the help of equations (16) – (19) within $\mu = \text{const}$, in side – within $z_1 = \text{const}$, $z_2 = \text{const}$. If to put $\varphi_1 = \text{const}$, $\varphi_2 = \varphi_1 / u$ (u – transfer number of transmission), then these equations determine coordinates of immediate contact lines of the working surfaces of pinion teeth and gear. Within $\lambda = \text{const}$ equations (16) – (19) determine dint of contact dot on teeth surface within its movement from one side to another. In common case equations (16) – (19) within $\lambda = \text{const}$ are equations of helical lines of variable pitch. Cylinder radiuses, on which these helical lines are placed from the first two equations (16) – (19), are equal

$$R_{1b} = \sqrt{(f_1 + \xi + R_1)^2 + (\Omega_1 \cos\beta)^2} ,$$
$$R_{2b} = \sqrt{(f_1 + \xi - R_2)^2 + (\Omega_1 \cos\beta)^2} . \qquad (20)$$

These equations can be used within the determination of toothing field borders, appropriate to the tops of pinion teeth and gear. It is necessary to put in (20) $R_{1b} = R_{a1}$, $R_{2b} = R_{a2}$ (R_{a1}, R_{a2} – radiuses of pinion teeth tops and gear) for it, and within the determination of μ to use meanings z_1 and z_2 from equations (16) – (19). Meanings z_1 and z_2 are necessary to be given within the width of teeth gears.

Within this case we will differ two types of teeth gears with arched teeth: gears with symmetric arched teeth (teeth are symmetric relatively to the plane XOZ (fig. 4) and asymmetric arched teeth (working area of teeth surface is situated from one side of plane (fig. 4).

In the first case meanings z in equations (14) and (15) are changed within $-B \le z \le B$ (where B – width of the teeth arc), in the second one – $B_1 \le z \le B$ (where B_1 – distance up to the nearest tooth side from plane XOZ (fig. 4).

Conclusions. Received results can be used within the determination of indices of loading ability and other characteristics of cylindrical arched transmissions with generalized teeth geometry within initial profile offset.

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ГЕОМЕТРІЯ ЗУБЦІВ АРОЧНИХ ПЕРЕДАЧ ПРИ ЗМІЩЕНІ ВИХІДНОГО КОНТУРА

Досліджена геометрія зубців циліндричних зубчастих коліс, нарізаних рейковим інструментом, зубці яких профільовані довільною кривою в нормальному поперечному перерізі та в прямолінійному напрямку в межах початкового зміщення профілю. З'єднання інструментальної рейки з дугоподібними зубцями із зубчастими колесами в межах початкового зміщення профілю є аналогом процесу нарізання зубців за допомогою методу обкочування. Поверхні зубців рейкового інструменту обкочують поверхні зубців зубчастих коліс. Якщо поверхні зубців інструментальної рейки для нарізання зубчастих коліс та шестерен невідповідні, то у наявний випадок точкового зачеплення шестерні та колеса. Якщо початкові профілі несуперечливі, то контактна точка переміщується з однієї сторони зуба на іншу. Було отримано рівняння поверхонь зубців, записуючи координати поверхонь зубців в системи координат $X_1Y_1Z_1$ *і* $X_2Y_2Z_2$*, пов'язані із шестірнею та колесом. Здійснюючи такий перехід, отримано рівняння поверхонь опуклої сторони зубців шестерні та увігнутої сторони зубців колеса. У загальному випадку ці рівняння є рівняннями гвинтових ліній змінної висоти. Ці рівняння можуть бути використані при визначенні меж границь зубців, відповідно шестерні та колеса. У цьому випадку два типи зубчастих коліс із дугоподібними зубцями будуть нарізані: зубчасті колеса із симетричними дугоподібними зубцями (зубці симетричні відносно площини XOZ та асиметричні дугоподібні зубці (робоча зона поверхонь зубців розташована з одного боку площини). Отримані результати можуть бути використані при визначенні показників навантажувальної здатності та інших характеристик циліндричних арочних передач із узагальненою геометрією зубців у межах початкового зміщення профілю.*

Ключові слова: *початковий профіль; твірна поверхня; арочні передачі; зміщення профілю; зубчаста рейка.*

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