

Numerical solution of hyperbolic equations by means of the package MathCAD

Svetlana Makovetska, Olga Seidykh

National University of Food Technologies, Kyiv

Mathematical package MathCAD is popular among mathematicians who are non-programmers, primarily due to its interface that approximates in the best way the look of MathCAD documents to traditional calculations "on paper". Package MathCAD is one of the common package containing "math" in the amount of engineering university and provides the user with a great set of tools for the implementation of numerical methods for solving mathematical problems with a computer.

Typically, the process of solving specific partial differential equations is a painstaking scientific work that requires extensive training of the researcher. Here the benefits package MathCAD may be fully revealed. Let's illustrate this in the report with a rather simple example of solving the wave equation using MathCad package.

Let's consider the one-dimensional wave equation that describes, for example, the free vibrations of the string from the output of non-equilibrium state. This movement can be observed when a string fastened at both ends is pulled from straight position and let loose in the further fluctuations free of force impact. The dynamics of displacement profile on a straight-line position of the string is described the wave equation:

$$\frac{\partial u^2}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$$

where $u(x, t)$ - lateral shift of the string with coordinate x at time t ; c - constant of the material of construction of the string.

Terms of fixing the ends of the strings of length l (boundary conditions) are: $u(0, t) = u(l, t) = 0$. The initial conditions set the shape of the string at the initial time: $u(x, 0) = f(x)$, and the output velocity distribution can be written as

$$\frac{\partial u(x,0)}{\partial t} = g(x)$$

Wave equation is one-dimensional and belongs to the hyperbolic type. Thus, the solution can be found using the function pdsolve. To do this, rewrite it as a system of two equations in partial derivatives of standard methods when an additional function is introduced $v(x, t)$, $v = \frac{\partial u}{\partial t}$.

$$\begin{cases} \frac{\partial v}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2} \\ \frac{\partial u}{\partial t} = v \end{cases} \quad 0 < x < l, \quad 0 < t < t_{\max}$$

$$u(0,t) = u(l,t) = 0, \quad u(x,0) = f(x), \quad \frac{\partial u(x,0)}{\partial t} = g(x)$$

Example.

Find the solution to the wave equation $\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}$, if the string's length is $l=2\pi$, and the string's initial form is $u(x,0) = \sin(\pi \cdot x/l)$, $t \in [0, 2\pi]$, $u(0,t) = u(l,t) = 0$.

Solution in MathCAD:

$$\underline{c} := 3 \quad \underline{L} := 2 \cdot \pi \quad \underline{T} := 2 \cdot \pi$$

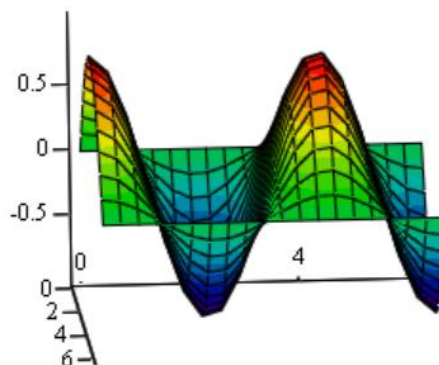
Given

$$v_t(x,t) = c^2 \cdot u_{xx}(x,t) \quad u_t(x,t) = v(x,t)$$

$$u(x,0) = \sin\left(\frac{\pi \cdot x}{L}\right) \quad v(x,0) = 0 \quad u(0,t) = 0 \quad u(L,t) = 0$$

$$\begin{pmatrix} u \\ v \end{pmatrix} := \text{Pdesolve}\left[\begin{pmatrix} u \\ v \end{pmatrix}, x, \begin{pmatrix} 0 \\ L \end{pmatrix}, t, \begin{pmatrix} 0 \\ T \end{pmatrix}\right]$$

$$M := \text{CreateMesh}(u, 0, L, 0, T)$$



The differential equations' solutions should be provided a visual and convenient form for the purposes of analysis. Solutions' visualization is primarily important for the results obtained by numerical methods, which have the appearance of numerical sets.

Problem solving using MathCAD allows the user to greatly enhance intellectual activity, the ability to automate calculations, clearly visualize the calculations' results thanks to the powerful graphics.

Literature

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