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BOUNDARY INTEGRAL EQUATIONS FOR PROBLEMS ABOUT PLANE DEFORMATIONS OF LINEAR VISCOTLASTIC MEDIUM

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The technological processes of food productions are often related to materials or raw material, mechanical properties of which are viscoelastic. In a plane viscoelastic area $D(t)$, limited by the smooth reserved contour $L(t)$ at $t \geq 0$ integro-differential equation of equilibrium is

$$\mu\Delta\vec{u}(\vec{y},t) + (\lambda+\mu)graddiv\vec{u}(\vec{y},t) - \mu\int_0^t q(t-\tau)[\Delta\vec{u}(\vec{y},\tau)+\frac{1}{3}graddiv\vec{u}(\vec{y},\tau)]d\tau + \vec{f}(\vec{y},t;\vec{u}) = \vec{0} \quad (1)$$

at the set tensions $\vec{p}_n(\vec{x},t)$ in contour point $L(t)$. Explanatory notes: μ,λ are instantly-resilient steelg; Δ is Laplace operator; $\vec{u}(\vec{y},t)$ is a displacement vector; $\vec{f}(\vec{y},t;\vec{u}) = \rho_0\vec{m}(\vec{y},t)[1-div\vec{u}(\vec{y},t)]$ ($\vec{m}(\vec{y},t)$ is mass force intensity, $\rho_0 = \rho(\vec{y},0)$ is material density, $\vec{y}\in D(t)$); $q(t) = ce^{-\beta t}t^{\alpha-1}$ is Rzhanicyn relaxation kernel ($\beta, c > 0,\ \alpha \in (0,\ 1)$ are parameters of material); $\vec{n}$ is a normal of the given contour point $\vec{x}\in L(t)$.

The solution of this problem is as a sum of partial solution of equation (1) and viscoelastics potentials of a simple layer: $\vec{u}(\vec{y},t) = \vec{u}[\vec{f}] + \sum_{k=1}^{2}\vec{e}^{\,k}\int_0^t d\tau \int_{L(\tau)} \vec{v}(l,\tau)\cdot\vec{v}^{(k)}(\vec{y}-\vec{x};t-\tau)dl$, (2)

where $\vec{v}^{(k)}(\vec{y}-\vec{x};t-\tau)$ is a fundamental solution of equation (1).

The substitution of expression (2) in a boundary condition results in the system of the second type integral equation in relation to a component of the sought vectorial density of potential $\vec{v}(l,t)\in L(t)$:

$$\pi v_1(l_0,t) + \int_{L(t)}\sum_{i=1}^{2} v_i(l,t)K_{1i}(l,l_0;t)\left|\frac{\partial\vec{x}(l,t)}{\partial l}\right|dl + \int_0^t \tilde{k}(t-\tau)d\tau \int_{L(\tau)}\sum_{i=1}^{2} v_i(l,\tau)k_{1i}(l,l_0;t,\tau)\left|\frac{\partial\vec{x}(l,\tau)}{\partial l}\right|dl = \psi_1(l_0,t); \quad (3)$$

$$\pi v_2(l_0,t) + \int_{L(t)}\sum_{i=1}^{2} v_i(l,t)K_{2i}(l,l_0;t)\left|\frac{\partial\vec{x}(l,t)}{\partial l}\right|dl + \int_0^t \tilde{k}(t-\tau)d\tau \int_{L(\tau)}\sum_{i=1}^{2} v_i(l,\tau)k_{2i}(l,l_0;t,\tau)\left|\frac{\partial\vec{x}(l,\tau)}{\partial l}\right|dl = \psi_2(l_0,t),$$

where $K_{ij}(l,l_0;t)$ and $k_{ij}(l,l_0;t,\tau)$ are equation kernel; $\tilde{k}(t)$, $\psi_1(l_0,t)$ and $\psi_2(l_0,t)$ are the known functions. The method of "steps at times" is used for numerical calculations of the proved system of integral eguafion of the 2-nd type (3).

KEY WORDS *Viscoelasticity, relaxation kernel, viscoelastic potential, fundamental solution, potential density, integral equation kernel.*

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