MODELING OF CUTTING OF MULTILAYER MATERIALS

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Abstract.

Introduction. *In the food industry products which have homogeneous or difficult multilayer structure are cut. Process of cutting for each case has many differences. It is necessary to fix features of movement of a knife in different products behind structure, and to improve regime parameters of operation of the cutting equipment.*

Materials and methods. *Mathematical simulation of movement of a knife in a product on the basis of the power assaying and the decision of differential second-kind equations is spent. For experimental researches installation of pendulum type with a knife that allows to change easily rate of a knife and a store of its kinetic energy is used.*

Results. *Mathematical sample pieces of cutting which allow to define force of cutting at different rates of a knife and product structure, and techniques of definition of factors, in mathematical sample pieces of cutting for different conditions of process are developed. If the product has a thin strong envelope, the force of cutting at knife approach to an envelope grows. It is reduced recommendations concerning conditions of movement of the cutting instrument, its orientation, concerning stratums to a product. At first it is necessary to cut a thin strong stratum, and then all volume of a product. It considerably reduces energy consumptions at cutting.*

Keywords: cutting, food, cutting regime, mathematical modeling, multi-layer, products.

I. Introduction.

In the food-processing industry machining job by cutting is applied to products which can be homogeneous for all volume (a sugar beet, the salted pork fat, meat without bones, confectionery masses but other), and for nonhomogeneous products which have an impregnation or an envelope. Structurally mechanical properties of an envelope differ from a product great bulk (fig. 1). For example, it is meat products which have interlayers from sinewy fabrics, a skin and bones; vegetables which have the strong outer heath; grain articles which consist of pulp and a strong rim. Stratums of these products are strong linked among themselves, and have different strength.

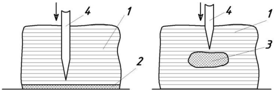


Fig. 1. Cutting of products with nonhomogeneous structure:

a - a product with an envelope; b - a product with an impregnation; 1 - a product, 2 - an envelope; 3 - an impregnation; 4 - a knife.

Analytical existing sample pieces of a cutoff have displayed that presence of impregnations or an envelope in a product it was not considered at simulation and optimisation of process of cutting.

Experimental researches prove [1- 4] that cutoffs of products with an envelope considerably differs from a cutoff of homogeneous products. At knife approach to an envelope there is a short-term increase in force of cutting, loads on a knife becomes pulsing, abrasion resistance of a knife decreases, quality of a surface of a cut is aggravated.

The envelope it can be on an input or an exit of a knife from a product, or to take place inside.

For definition of regularities of cutting of nonhomogeneous products on structure are spent analytical and experimental researches.

II. Materials and methods.

The technique of analytical probes assumes:

- The assaying of movement of a knife in a product under the influence of the affixed forces
- Working out of mathematical sample pieces of movement of a knife in a product on the basis of differential second-kind equations
- Definition of factors of mathematical sample pieces of a cutoff
- Practical recommendations about application of sample pieces for definition of force of cutting and a choice of rational parameters of process

The technique of experimental researches assumes use idle time on a construction, but with ample opportunities of application of the trial type. It is a floating lever [1, 3] (fig. 2) with a knife which at movement cuts a product.

Rate of a knife and its kinetic energy easily vary in considerable breaking points by balance start under a different edge and change of positions of a weight 3.

Rate of a knife it is defined on the basis of the differential equations of movement of a balance. The formula is as a result received:

$$V = R\sqrt{2\frac{\sum P_i r_i}{J}(1-\cos\beta)}$$

P_i - weight of each detail of a balance; R_i- distance from a detail barycentre to a shaft fulcrum pin; β - an edge of start or balance carriage, R - length of a balance; J - moment of inertia of all details of a balance.

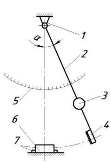


Fig. 2. The device for cutting process research:
1 - plate; 2 - balance; 3 - load; 4 - razor; 5 - scale; 6 - product; 7 - product fixator.

We define rate of an input and an exit of a knife from a product at various edges α balance start. The received experimental data we define forces of cutting and factors of mathematical sample pieces for various products on properties.

III. Result and discussing.

1. Simulation of a cutoff of homogeneous products

In case of product cutting, has homogeneous structure on all volume, on a knife forces operate:

• Cut F_r

• Friction (a dynamic friction)

$$G = C + k_1 V = C + k_1\frac{dy}{dt}$$

• Inertia

$$P_i = ma = \frac{m d^2 y(t)}{dt^2}$$

k_1 - the factors characterizing the friction; V, a - rate and knife speedup in a product.

Let's record the differential equation of movement of a knife:

$$F_r + G + P_i = 0 \Rightarrow \quad F_r + (C + k_1\frac{dy(t)}{dt}) + m\frac{d^2y(t)}{dt^2} = 0 \qquad (1)$$

We solve the equations and it is spent some transformations. We receive the equation for definition of force of cutting depending on rate of a knife and properties of a product:

$$F_r = \frac{k_1\frac{dy(t)}{dt} - e^{-\frac{k_1 \cdot t}{m}}(C + V_{oy}k_1) + C}{e^{-\frac{k_1 \cdot t}{m}} - 1} \qquad (2)$$

The detailed assaying of the equation (2) and received results it is displayed in operation [3].

2. Simulation of a cutoff of nonhomogeneous products

Feature of a cutoff of nonhomogeneous products which have impregnations or an envelope, is short-term (instant) growth of force of cutting.

For such products of the equation of movement of a knife:

$$F_r + G + P_i + F_M = 0 \qquad (3)$$

Fm - instant forces.

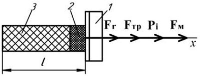


***Fig. 3.** The forces operating on a knife:*
1 - a knife, 2 - an envelope, 3 - a product.

If impregnations is not on a product surface, and in its interior (fig. 1), the differential equation of movement of a knife looks like:

$$F_r + (c_1 x + \mu_1\frac{dy(t)}{dt}) + m\frac{d^2y(t)}{dt^2} + F_M = 0 \qquad (1)$$

Where FM - quickly operating (instant) forces;

$$F_M = ke^{-(t-a)^n} \qquad (2)$$

k - the factor characterizing resistance of a cutoff of an impregnation. Its value depends on strength of an impregnation; a - the factor, defined impregnation arrangements in a product.

On fig. 4 influence of factors k and an on forces and an impregnation arrangement in a product is displayed.

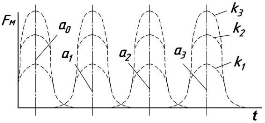


Fig. 4. *Impact of factors k and a on instant forces and an impregnation arrangement in a product; a3> a2> a1, k3> k2> k1*

The sizes of an impregnation are influenced by an apparent exponent n (fig. 5)

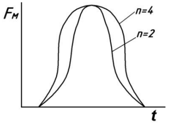


Fig. 5. *Influence of magnitude of an apparent exponent n on the sizes of an impregnation.*

Let's substitute F_M in the equation (1) and we will receive its decision. Under entry conditions t = 0 = x (0) = 0, V (0) = V0:

$$x(t)=\frac{m(F_p+c_1)(1-e^{-\frac{\mu_1 t}{m}})}{\mu_1^2}+\frac{1}{2}(2mV_0(1-e^{-\frac{\mu_1 t}{m}})+\sqrt{\pi}(erf(-t+a)-erf(a)+$$
$$+e^{(-1/4\frac{\mu_1(-4am-\mu_1+4tm)}{m^2})}(erf(\frac{1}{2}\frac{\mu_1}{m}+a)-erf(-t+\frac{1}{2}\frac{\mu_1}{m}+a)))k-2t(F_p+c_1))/\mu_1 \quad (2)$$

$$V=\frac{(F_p+c_1)e^{-\frac{\mu_1 t}{m}}}{\mu_1}+(V_0\mu_1 e^{-\frac{\mu_1 t}{m}}+(-e^{-(-t+a)^2}-\frac{\sqrt{\pi}\mu_1 e^{(-\frac{\mu_1(-4am-\mu_1+4tm)}{4m^2})}(erf(\frac{1}{2}\frac{\mu_1}{m}+a)-erf(-t+\frac{1}{2}\frac{\mu_1}{m}+a))}{2m}+$$
$$+e^{-\frac{\mu_1(-4am-\mu_1+4tm)}{4m^2}}e^{(-(-t+1/2\frac{\mu_1}{m}+a)^2)})k-F_p-c_1)/\mu_1 \quad (3)$$

From the equation (3) we define force of cutting:

$$F_p=\frac{1}{2}(2Vm\mu_1-2e^{-\frac{\mu_1 t}{m}}mc_1-2V_0\mu_1 e^{-\frac{\mu_1 t}{m}}m+2ke^{(-(-t+a)^2)}m+k\mu_1 e^{(\frac{\mu_1(4am+\mu_1-4tm)}{4m^2})}\sqrt{\pi}(erf(\frac{\mu_1+2am}{2m})-$$
$$-erf(\frac{-2tm+\mu_1+2am}{2m}))-2ke^{\frac{\mu_1(4am+\mu_1-4tm)-(-2tm+\mu_1+2am)^2}{4m^2}}m+2c_1m)/(m(e^{-\frac{\mu_1 t}{m}}-1)) \quad (4)$$

The received equations are difficult for the analytical assaying, at execution of practical calculations and use of computer methods of character mathematics demand.

Let's consider possibilities of simplification of definition of force F_M.

If on a product surface the stratum is placed (an envelope, a rim, a packing film), the instant force can vary under such laws:

$F_M = Be^{-bt}$ (5)

or

$F_M = Ae^{-B(\ln t)^2}$ (6)

Change of instant force on time is displayed on fig. 6. Graphically it is figured as quickly falling down dependence. Time of its operation 0 - t1 is much less, than duration of movement of a knife in a product.

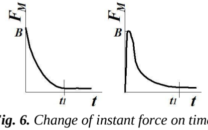


Fig. 6. *Change of instant force on time.*

At the differential equation of movement of a knife:

$$F_r + (C + k_1 \frac{dy(t)}{dt}) + m\frac{d^2 y(t)}{dt^2} + Be^{-bt} = 0 \qquad (7)$$

We solve the equations under entry conditions t = 0 = x (0) = 0, V (0) = V0 we receive a value of migration (8), rates of a knife (9) in a product and force of cutting (10).

$$y(t) = \frac{m(1-e^{-\frac{\mu_1 t}{m}})(V_0\mu_1 + C_{tp} + F_p)}{\mu_1^2} + \frac{\mu_1(B(1-e^{(bt)}) + bt(C_{tp} + F_p)) - mb(bt(C_{tp} + F_p) + B(1-e^{-\frac{\mu_1 t}{m}}))}{(mb-\mu_1)\mu_1 b} \qquad (8)$$

$$V(t) = \frac{e^{-\frac{\mu_1 t}{m}}(V_0\mu_1 + C_{tp} + F_p)}{\mu_1} + \frac{\mu_1(Bbe^{(-bt)} + b(C_{tp} + F_p)) - mb(b(C_{tp} + F_p) + \frac{B\mu_1 e^{-\frac{\mu_1 t}{m}}}{m})}{(mb-\mu_1)\mu_1 b} \qquad (9)$$

$$F_p = \frac{V\mu_1 mb - V\mu_1^2 - e^{-\frac{\mu_1 t}{m}}V_0\mu_1 mb + e^{-\frac{\mu_1 t}{m}}V_0\mu_1^2 - e^{-\frac{\mu_1 t}{m}}mbC_{tp} + e^{-\frac{\mu_1 t}{m}}\mu_1 C_{tp} - \mu_1 Be^{(-bt)} - \mu_1 C_{tp} + C_{tp}bm + B\mu_1 e^{-\frac{\mu_1 t}{m}}}{e^{-\frac{\mu_1 t}{m}}mb - e^{-\frac{\mu_1 t}{m}}\mu_1 + \mu_1 - mb} \qquad (10)$$

On fig. 7 change of instant force depending on factor is displayed in which strength and structurally-mechanical properties of an envelope influence.

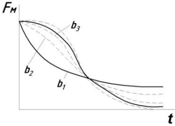


Fig. 7. *Dependence of instant force F_M of an apparent exponent of B*

We define power of cutting as product of force of cutting for rate of a knife.

$$N = F_{rm}\frac{dy(t)}{dt}$$

F_{rm} - the maximum force of cutting.

If instant force, $F_M = Ae^{-B(\ln t)^2}$ the equation of movement of a knife:

$$F_r + (C + k_1 \frac{dy(t)}{dt}) + m\frac{d^2 y(t)}{dt^2} + Ae^{-B(\ln t)^2} = 0 \qquad (11)$$

We solve the equations under the same entry conditions, we recognize forces of cutting:

$$F_r = \frac{\mu_1(V - e^{-\frac{\mu_1 t}{m}}V_0) - e^{-\frac{\mu_1 t}{m}}C + A\int_0^t \frac{\mu_1 e^{-\frac{b\ln U^2 m - \mu_1 t + \mu_1 U}{m}}}{m}dU - Ae^{b\ln(t)^2} + A\left(\frac{\partial}{\partial t} invlaplace\left(\frac{laplace\left(e^{b\ln(t)^2}, t, s_1\right)}{s_1}, t, s_1\right)\right) + C}{e^{\left(-\frac{\mu_1 t}{m}\right)} - 1} \qquad (12)$$

The equation powers up special functions, therefore it is expedient to execute its decision the approximate methods in the form of a polynomial or numerical methods.

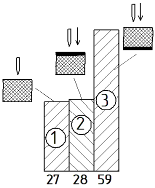
Change of force of cutting in the presence of an envelope it is confirmed experimentally. It is received forces of a cutoff of meat with a sinewy interlayer according to the equation (2). An interlayer took place on an input or an exit of a knife from a product. The force of cutting separately sinewy laminations small, also was not fixed by gears. But forces of a cutoff of meat increases at stratum arranging on an exit of a knife from a product (fig. 8).

Adequacy of the received sample pieces is confirmed for not foodstuff. For example, presence of a thin polymeric film on a cardboard augments forces of its cutoff at 20-50 time under condition of film arranging on a knife exit.

On the basis of the spent probes following outputs and recommendations are drawn:

The product structure influences force of cutting. Presence of an envelope and its arranging on an exit of a knife from a product (fig. 9a) considerably augments force of cutting, therefore so to cut a product of the irrational

At a cutoff the envelope should take place on a knife input in a product, or is perpendicular to a knife (fig. 9,). At execution of this condition the force of cutting decreases, quality of a cut and a period of operation of the cutting instrument raises.



Specific force of cutting, *kN / m*

Fig. 8. *Dependence of force of cutting of meat on arranging sinewy a stratum:*
1 - without an interlayer, 2 - a stratum on a knife input in a product, 3 - a stratum on an exit of a knife from a product.

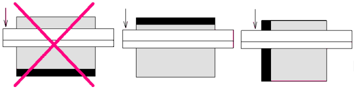


Fig. 9. *Arranging of an envelope concerning knife movement.*

IV. Conclusions

Process of a cutoff of products, homogeneous or have an envelope or an impregnation, considerably differs.

At knife approach to an envelope of force of cutting it is augmented multiply under the exponential law.

Mathematical sample pieces of movement of a knife are developed allow to define force of cutting for various products on structure.

At a cutoff of multilayer products at first it is necessary to cut a thin strong stratum. It allows to lower expenditures of energy for cutting process.

Results can be applied at a choice of conditions and conditions of cutting of foodstuff and many other materials, in particular, the packing.

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