

ON ESTIMATES OF SINGULAR NUMBERS OF A HILBERT–SCHMIDT INTEGRAL OPERATOR

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In $L_2[0; 1]$, let us consider the integral operator

$$(Af)(t) := \int_0^1 a(t, s)f(s) ds, \quad f \in L_2,$$

with a Hilbert–Schmidt kernel, i.e., $a(t, s)$ is a square integrable function measurable on $[0; 1] \times [0; 1]$

$$\int_0^1 \int_0^1 |a(t, s)|^2 dt ds < \infty.$$

As usual, $\lambda_k(A)$ denote the eigenvalues of the operator A enumerated in the order of nonincreasing of their moduli; $s_k(A) = \sqrt{\lambda_k(AA^*)}$ are singular numbers (s -numbers) of the operator A , where A^* is the operator adjoint to A . Consider the case where the domain of values of the operator A belongs to a space of continuous functions. To this end, we introduce the notion of a continuity modulus of order $m = 1, 2, \dots$ for the kernel $a(t, s)$. For each $0 \leq \delta \leq m^{-1}$, we set

$$\begin{aligned} \omega_m(\delta, a) := \\ \sup_{f \in L_2, \|f\|_2=1} \sup_{0 \leq h \leq \delta} \sup_{0 \leq t \leq 1-hm} \left| \sum_{q=0}^m (-1)^q \binom{m}{q} (Af)(t + hq) \right|. \end{aligned} \quad (1)$$

For $\delta > m^{-1}$ we consider that $\omega_m(\delta, a) = \omega_m(m^{-1}, a)$.

Theorem 1. *Let the domain of values of the operator A belong to a space of continuous functions, and $m = 2, 3, \dots$*

Then

$$\sum_{k=r}^{\infty} s_k^2(A) \leq 25\omega_m^2 \left(\frac{2}{r}, a \right), \quad r = m + 1, m + 2, \dots$$

This theorem yields the following proposition.

Corollary. *Let the domain of values of the operator A belong to a space of continuous functions, and $m = 2, 3, \dots$*

Then

$$s_r(A) \leq \left(\frac{10m}{r} \right)^{\frac{1}{2}} \omega_m \left(\frac{4}{r}, a \right), \quad r = m + 1, m + 2, \dots$$