

Synthesis of modal regulators with an observer of the Luenberger

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For a controlled object described as

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t), \\ y(t) = Cx(t), \end{cases} \quad (1)$$

(where $x(t) \in R^n$ – the state of the regulator, $u(t) \in R^m$ – control, $y(t) \in R^p$ – the measured output of the object), we choose a regulator in the form of an observer of the state of Luenberger of complete order

$$\begin{cases} \dot{x}_r(t) = Ax_r(t) + Bu(t) + L(Cx_r(t) - y(t)), \\ u(t) = Kx_r(t), \end{cases} \quad (2)$$

where $x_r(t) \in R^n$ – condition of the regulator.

The matrices K and L for the closed system (1), (2) are defined in the *LMI*-domain.

We introduce a vector of discrepancy $e(t) = x(t) - x_r(t)$ and, as a state of a closed system, we choose a vector $(x^T(t), e^T(t))^T$ satisfying the generalized equation

$$\frac{d}{dt} \begin{pmatrix} x(t) \\ e(t) \end{pmatrix} = \begin{pmatrix} A + BK & -BK \\ 0 & A + LC \end{pmatrix} \begin{pmatrix} x(t) \\ e(t) \end{pmatrix}.$$

For D -stability of system (1) and (2) it is necessary that the matrices $A + BK$ and $A + LC$ are D -stable [1]. For the matrix $A + BK$, we arrive at the form of *LMI*

$$\begin{aligned} M(A + BK, X_1) &= P \otimes X_1 + G \otimes ((A + BK)X_1) + G^T \otimes (X_1(A + BK)^T) = \\ &= M(A, X_1) + G \otimes (BX_1) + G^T \otimes (X_1^T B^T) < 0, \end{aligned} \quad (3)$$

(where $Z_1 = KX_1$), and for the matrix $A + LC$ we get one more *LMI*

$$\begin{aligned} L(A + LC, X_2) &= P \otimes X_2 + G \otimes (X_2(A + LC)) + G^T \otimes ((A + LC)^T X_2) = \\ &= L(A, X_2) + G \otimes (Z_2 C) + G^T \otimes (C^T Z_2^T) < 0, \end{aligned} \quad (4)$$

where $Z_2 = X_2 L$.

For the stability of the object (1) it is sufficient [2] that the linear matrix inequalities (3) and (4) are solved with respect to the variables $X_1 = X_1^T > 0$, Z_1 and $X_2 = X_2^T > 0$, Z_2 . Then the settings for the controller are as follows $K = Z_1 X_1^{-1}$, $L = X_2^{-1} Z_2$.

Literature

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