The mathematical simulation of heat-mass transfer in bread baking process

Volodymyr TELYCHKUN[1], Evgeny SHTEFAN[1], Yuliya TELYCHKUN[1]

Abstract. *The problem of mathematical model designing of the heat-mass transfer processes in bread baking is considered. The analytical part of mathematical model (analytical model) is submitted. The mathematical model is focused on the modern computer technologies using.*

Keywords: *mathematical model, process of heat-mass transfer, multicomponential system, analytical model*

Introduction

The baking industry enterprises uses a variety of furnaces, which differ among themselves by productivity, design features, heat distribution system, operation stability, bread products quality etc. The modern furnace design development is based on the great practical experience in the design and operation of the equipment with the experimental and theoretical studies of the baking processes using [1,2]. Therefore a theoretical basis for the processes that occurs during "dough workpiece – bread product" is the actual scientific problem. At the same time the latest research analysis shows that the reasonable calculation methods for optimal mode definition of baking equipment are absent. Existing design methods for baking furnace are based on productivity, heat and fuel calculations without taking in account the internal heat processes and bread quality indicators (color, luster, shape, size, taste, smell, etc.) creation [1,4]. Such calculations could be carried out by the way of complete and accurate mathematical description of each of the processes that occur during bread baking. So, mathematical simulation of the baking process may be the base for the optimal process conditions substantiation and allow to formulate the requirements for thermal regime and furnace construction.

Materials and methods

The proposed methods for construction and technological parameters of baking prosseces are based on information practice of design (IPD) [5]. IPD has the type: "mathematical model - intellectual expert system - design automation system" and considers the baking processes as multicomponential system of interconnected subjects of inquiry: dough workpiece (DW), technological equipment elements, thermo- mechanical loading etc. Schematically IPD represented in Figure 1. The functional basis of IPD is the mathematical model of heat-mass transfer in the DW under specified conditions of heating. At construction of the analytical part of the mathematical model of the "DW – bread product" transformation we are guided by a principle of its conditional division on three groups: 1 - solid particles; 2 - water in various kinds and conditions; 3 - gaseous inclusions. Dough workpiece is considered as the moisture contained dispersed system with concrete geometrical parameters.

Results and discussions

One of the main parameter which characterizes the moisture transfer process is the mass content [1]:

$$\alpha_m(\mathrm{t}) = M_{\ae}(t) / \grave{I}_{\eth} \qquad (1)$$

or volume material content:

$$\alpha_v(t) = V_{\ae}(t) / V_{\eth} \qquad (2)$$

where $M_ж(t)$ – the liquid phase mass of the volume $V_ж$ (t) in the disperse material representative element with the volume $V_т$; $M_т$ (t)- - mass of the porous skeleton (solid phase). The moisture in the porous skeleton can be located in a liquid or gaseous states (depending on the temperature). Moisture content changes occur as a result of redistribution in the volume of the porous material (diffusion mechanism) with the possibility of going beyond borders through the surface of the DW. The DWtemperature determines not only by the physical state of moisture, but also by the thermodynamic forces, which realize the transfer of heat and moisture:

$$F_t = -\frac{1}{T} gradT$$
$$F_u = -T grad\left(\frac{\eta}{T}\right) \qquad (3)$$

where -diffuse potential [6], T - temperature.

The density of heat and moisture fluxes are defined by Onsager linear principle [6]:

$$J_t = -\frac{L_{11}}{T} gradT - L_{12} T\ grad\left(\frac{\eta}{T}\right) \qquad (4)$$

$$J_u = -\frac{L_{21}}{T} gradT - L_{22} T\ grad\left(\frac{\eta}{T}\right) \qquad (5)$$

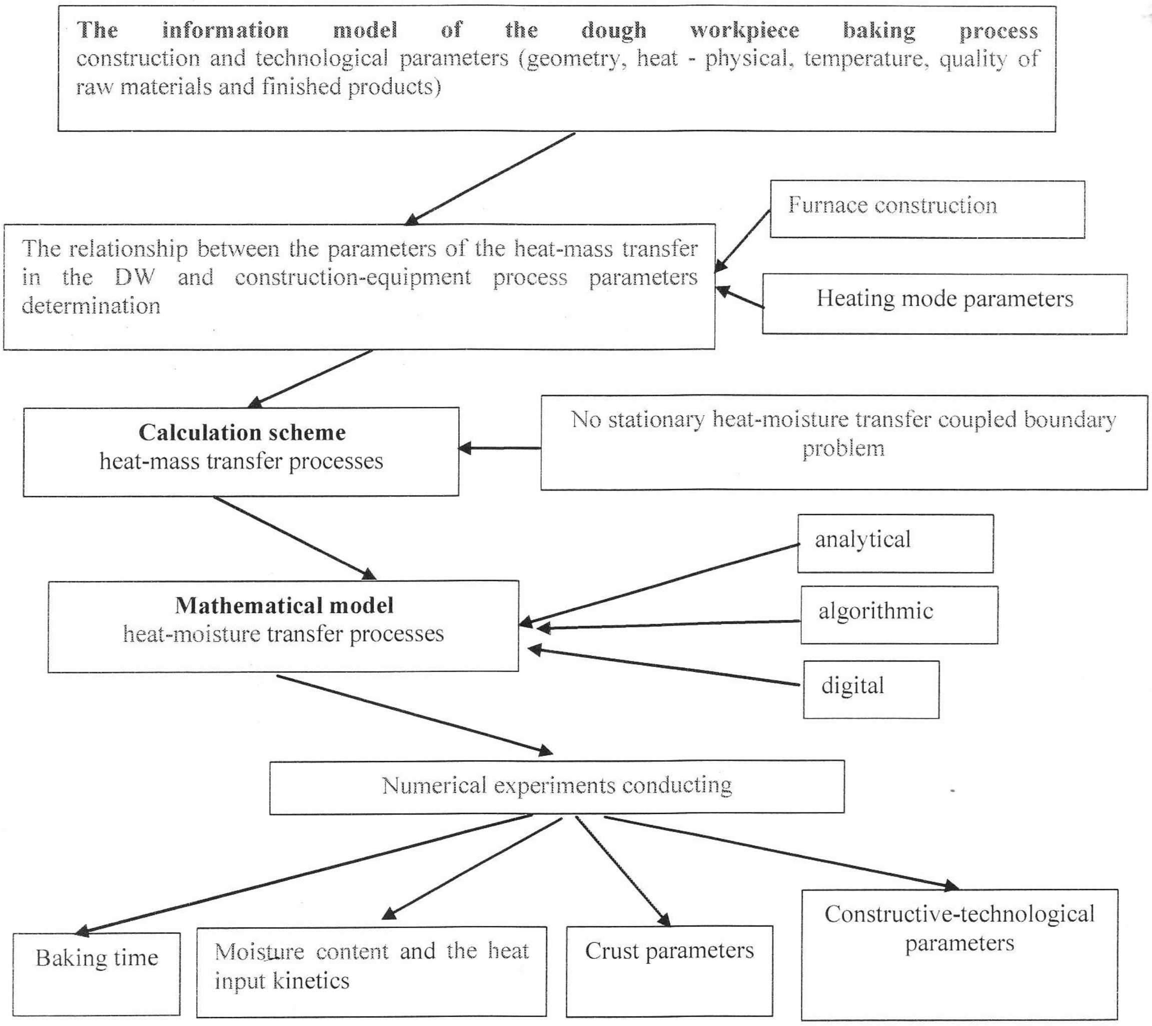

Figure 1. *Scheme for the structural and technological parameters of the bread baking process determination*

From (4) and (5) follows, that the heat flux is determined not only by the temperature gradient but by the potential diffusion gradient also. Similarly the mass of the moisture flow determined not only by the capacity of the diffusion gradient but by temperature gradient (thermal diffusion) also. Assuming the principle of reciprocity $L_{12} = L_{21}$ heat-moisture transfer process in disperse material may be defined by the generalized state vector:

$$\{Y\} = \begin{Bmatrix} T \\ u \end{Bmatrix} \qquad (6)$$

where u - moisture mass per unit volume.

Using (6) the process moisture transfer is described by the equations [2]:

$$\frac{\partial Y}{\partial t} = A\,div\,grad\,Y + D\,div\,grad\,Y^{-1} + W \qquad (7)$$

where $Y^{-1} = \begin{Bmatrix} u \\ T \end{Bmatrix}$; (8)

$$A = \begin{Bmatrix} a_t \\ a_m \end{Bmatrix}; \qquad (9)$$

a_t - the temperature – conductivity coefficient:

$$a_t = \frac{\lambda}{c_p \rho}, \qquad (10)$$

where λ - the material thermal conductivity coefficient.

For moisture contained material

$$\lambda = \alpha_г \lambda_г + \alpha_m \lambda_m;\ \alpha_г + \alpha_m = 1, \qquad (11)$$

where $\alpha_г$, $\alpha_т$ - the volume contents of the gas and solid phases respectively; $\lambda_г$ - the gas thermal conductivity coefficient; $\lambda_т$ - the solid phase material thermal conductivity coefficient:

$$\lambda_т = \lambda_0 + c(T - T_0), \quad (12)$$

where, λ_0, c, T_0- constants.

The heat capacity of the material c_p is determined as:

$$c_p = \alpha_г c_г + \alpha_m c_m; \quad (13)$$

where $c_г$ - the gas phase heat capacity coefficient; c_m - the solid phase heat capacity coefficient:

$$c_m = c_0 + d(T - T_0), \quad (14)$$

where c_0, d, T_0 - constants.

The disperse material density:

$$\rho = \alpha_г \rho_г + \alpha_т \rho_т; \quad (15)$$

$\rho_г, \rho_т$ - the gas and solid phases densities respectively; $\rho_г$ - accepted as a constant; $\rho_т = \rho_0 - b(T - T_0)$, ρ_0, b, T_0 - constants; a_m - the moisture diffusion coefficient.

$$D = \begin{Bmatrix} d_t \\ d_m \end{Bmatrix}; \quad (16)$$

d_t - the diffusive thermal conductivity coefficient:

$$d_t = \frac{d_m T}{C_{ж} \alpha_m} \left(\frac{\partial \eta}{\partial u}\right); \quad (17)$$

d_m - the thermal diffusion coefficient: $d_m = a_m \delta_u$, where δ_u - the relative molecular moisture flow coefficient:

$$W = \begin{Bmatrix} W_T \\ W_u \end{Bmatrix}; \quad (18)$$

W_T - specific power of the internal heat sources:

$$W_T = W_s + W_q; \quad (19)$$

$$W_s = \varepsilon \frac{C_m}{C_q} \frac{\partial u}{\partial t} \quad (20)$$

the source due to the "vapor-liquid" transformation; ε - phase transformation coefficient, which is determined from the experimental data:

$$\varepsilon = \exp(-0{,}138(100 - T)) \text{ at } T \leq 100^0C;$$

$$\varepsilon = 1 \text{ at } T \geq 100^0C; \quad (21)$$

W_q - source due to the different physical mechanisms; W_u - moisture source specific power.

For the phase transformations modeling of the DW nonsteady heating the following criteria used:

1. Moisture according "vapor-liquid" system – temperature condition $T \geq 100^0C$. The moisture physical state is determined by the appropriate set of thermophysical characteristics and phases volume contents according (1), (2),(11),(13).

2. The dispersed material solid phase according "dough-crumb-crust" system. This transformation is simulated by the moisture content changing in each element of the considered area:

a) "crumb" $u_k \leq u \leq u_т$, where $u_т$ – the dough moisture content, u_k – the "crust" moisture content;

b) "crust" $u \leq u_к$;

For the system of equations (1) - (21) closing we must supplement them by initial and boundary conditions:

1. At t = 0 the distribution of the parameter Y(0) = Y_0(X) are given.

2. On the DW boundary part the parameter Y_1(X, t)) are given.

3. On the DW boundary part thermal mass flow are defined:

$$\frac{\partial Y}{\partial n} = \varphi_2(X, t). \quad (22)$$

4. On the DW boundary part vector $Q = \beta(Y - Y_0)$) is given (means heat-mass transfer), β - the heat-mass transfer coefficient.

Conclusions

Equations (1) - (22) are consist the analytical model of DW baking processes. The future investigation will be devoted to the development of the methods for presented equations solving (algorithmic model) and the application of computer technology to effectively caring out calculation experiments (digital model). So, presented mathematical relations are the basis for the creation of an automated high-performance system for dough-bread transformation regularities analysis.

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